THEORETICAL BACKGROUND TO THE ENGINEERING CRITICAL ASSESSMENT METHODOLOGY FOR REELED PIPE INCORPORATED IN THE COMPUTER PROGRAM FLAWPRO VERSION 3

SwRI® Project No. 18.10265

SwRI Program Manager: Stephen J. Hudak, Jr.

Prepared for

Joint Industry Project (JIP) titled “Validation of a Methodology for Assessing Defect Tolerance of Welded Reeled Risers”

Prepared by

G. Graham Chell
Materials Engineering Department
Southwest Research Institute®
San Antonio, Texas

December 31, 2006
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOMENCLATURE</td>
<td>ix</td>
</tr>
<tr>
<td>EXECUTIVE SUMMARY</td>
<td>xvi</td>
</tr>
<tr>
<td>1.0 INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2.0 STRESS ANALYSIS</td>
<td>4</td>
</tr>
<tr>
<td>2.1 Elastic Analysis</td>
<td>4</td>
</tr>
<tr>
<td>2.1.1 General</td>
<td>4</td>
</tr>
<tr>
<td>2.1.2 Determining Normalized Stress Variations Using BS 7910</td>
<td>6</td>
</tr>
<tr>
<td>2.2 Elastic-Plastic Analysis</td>
<td>10</td>
</tr>
<tr>
<td>2.2.1 General</td>
<td>11</td>
</tr>
<tr>
<td>2.2.2 Constant Strain-based Stress Analysis Related to Reeling</td>
<td>12</td>
</tr>
<tr>
<td>Determining the Spring-back Strain During Straightening After Unreeling</td>
<td>12</td>
</tr>
<tr>
<td>Determining the Reeling Residual Stress</td>
<td>13</td>
</tr>
<tr>
<td>2.2.3 Constant Load-based Stress Analysis Related to Installation and Service</td>
<td>14</td>
</tr>
<tr>
<td>General</td>
<td>14</td>
</tr>
<tr>
<td>Determining the Relaxed Residual Stress after Shakedown</td>
<td>15</td>
</tr>
<tr>
<td>3.0 STRESS INTENSITY FACTORS</td>
<td>17</td>
</tr>
<tr>
<td>3.1 Weight Function Method</td>
<td>18</td>
</tr>
<tr>
<td>3.1.1 General</td>
<td>18</td>
</tr>
<tr>
<td>3.1.2 Weight Function Formulation</td>
<td>19</td>
</tr>
<tr>
<td>3.2 Validation of the FlawPRO Weight Function Approach</td>
<td>20</td>
</tr>
<tr>
<td>3.2.1 Surface Flaws</td>
<td>20</td>
</tr>
<tr>
<td>3.2.2 Embedded Flaws</td>
<td>25</td>
</tr>
<tr>
<td>4.0 NET SECTION YIELD LOADS AND REFERENCE STRESSES</td>
<td>30</td>
</tr>
<tr>
<td>4.1 General</td>
<td>30</td>
</tr>
<tr>
<td>4.2 Reference Stresses for Combined Tension and Bend Loading</td>
<td>31</td>
</tr>
<tr>
<td>4.2.1 Embedded and Surface Flaws</td>
<td>31</td>
</tr>
<tr>
<td>4.2.2 Through-Wall Flaws</td>
<td>32</td>
</tr>
<tr>
<td>5.0 J ESTIMATION SCHEMES FOR REELING, INSTALLATION AND SERVICE</td>
<td>33</td>
</tr>
<tr>
<td>5.1 General</td>
<td>33</td>
</tr>
<tr>
<td>5.2 Effects of Shakedown (Elastic-Plastic Relaxation) on J</td>
<td>34</td>
</tr>
<tr>
<td>5.3 J Formulation for Load Controlled Situations</td>
<td>35</td>
</tr>
<tr>
<td>5.3.1 Elastic Component of J</td>
<td>35</td>
</tr>
<tr>
<td>5.3.2 Plastic Component of J</td>
<td>35</td>
</tr>
<tr>
<td>5.4 Validation of the FlawPRO J estimations Scheme for Flaws at Stress Concentration Features under Load Control</td>
<td>36</td>
</tr>
</tbody>
</table>
**TABLE OF CONTENTS (Continued)**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5 J Formulation for Strain Controlled Reeling</td>
<td>37</td>
</tr>
<tr>
<td>5.5.1 General</td>
<td>37</td>
</tr>
<tr>
<td>5.5.2 Elastic Component of J under Strain Controlled Loading</td>
<td>37</td>
</tr>
<tr>
<td>5.5.3 Plastic Component of J under Strain Controlled Loading</td>
<td>37</td>
</tr>
<tr>
<td>5.6 Validation of the FlawPRO J Estimation Scheme for Flaws in Pipes Subjected to Reeling</td>
<td>38</td>
</tr>
<tr>
<td>5.7 Effects of Stress Concentration Features on J during Reeling</td>
<td>41</td>
</tr>
<tr>
<td>5.8 $\Delta J$ Estimation Scheme for Strain-controlled Reeling</td>
<td>41</td>
</tr>
<tr>
<td>5.8.1 General</td>
<td>41</td>
</tr>
<tr>
<td>5.8.2 Elastic Component of $\Delta J$ Under Strain-controlled Reeling</td>
<td>42</td>
</tr>
<tr>
<td>5.8.3 Plastic Component of $\Delta J$ Under Strain-controlled Reeling</td>
<td>43</td>
</tr>
<tr>
<td>6.0 MECHANICS OF CRACK GROWTH</td>
<td>51</td>
</tr>
<tr>
<td>6.1 Cyclic Crack Growth</td>
<td>51</td>
</tr>
<tr>
<td>6.1.1 General</td>
<td>51</td>
</tr>
<tr>
<td>6.1.2 Fatigue Crack Growth (Reeling)</td>
<td>52</td>
</tr>
<tr>
<td>6.1.3 Fatigue Crack Growth (Installation and Service)</td>
<td>55</td>
</tr>
<tr>
<td>6.2 Ductile Tearing during Reeling</td>
<td>56</td>
</tr>
<tr>
<td>6.3 Tear-Fatigue during Reeling</td>
<td>57</td>
</tr>
<tr>
<td>6.4 Validation of the Memory Model Based Tear-Fatigue Model in FlawPRO</td>
<td>59</td>
</tr>
<tr>
<td>7.0 CRACK TRANSITIONING</td>
<td>61</td>
</tr>
<tr>
<td>7.1 General</td>
<td>61</td>
</tr>
<tr>
<td>7.2 Criteria for Initiating Crack Transitioning</td>
<td>61</td>
</tr>
<tr>
<td>7.3 Types of Transitions</td>
<td>63</td>
</tr>
<tr>
<td>7.4 Sizes of Flaws after Transitioning</td>
<td>63</td>
</tr>
<tr>
<td>7.5 Effects of Transitioning on Fatigue Crack Growth Rates</td>
<td>64</td>
</tr>
<tr>
<td>8.0 CONCLUSIONS</td>
<td>65</td>
</tr>
<tr>
<td>9.0 ACKNOWLEDGEMENTS</td>
<td>65</td>
</tr>
<tr>
<td>10.0 REFERENCES</td>
<td>66</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table | Page
----|-----
3.1  | Ranges of applicability of LawPRO SIF solutions ............................................................17
7.1  | Crack transitioning criteria used in FlawPRO .....................................................................61
7.2  | Ranges of applicability of FlawPRO SIF solutions .............................................................62
7.3  | Crack transitioning possibilities .........................................................................................63

LIST OF FIGURES

Figure | Page
-------|-----
2.1   | Schematic of the loads acting on welded pipes .................................................................4
2.2   | Illustration of how misalignment may result in geometrical changes to weld geometrical discontinuities assumed for aligned pipe .........................................................6
2.3   | Comparison of the BS 7910 magnification factors $M_k$ for 2-D flaws at weld geometrical discontinuities subjected to membrane and through-wall bend stressing with the variation in local stresses derived from these factors using the procedures in FlawPRO ........................................................................................................8
2.4   | Demonstration of the consistency of the FlawPRO calculated $M_k$ factors for flaws at weld geometrical discontinuities subjected to nominal membrane and through-wall bend stressing with the BS 7910 magnification factors ..........................................................8
2.5   | Typical $M_k$ factors for 3-D flaws at weld geometrical discontinuities subjected to nominal membrane stressing calculated using the weight function SIF routines in FlawPRO ........................................................................................................9
2.6   | Typical $M_k$ factors for 3-D flaws at weld geometrical discontinuities subjected to nominal through-wall bend stressing calculated using the weight function SIF routines in FlawPRO ........................................................................................................9
2.7   | Illustration of the cause of the singularity in the FlawPRO derived $M_k$ factor for deep semi-circular flaws subjected to through-wall bending .............................................9
2.8   | Examples of typical “smoothed” normalized local stress variations for nominal membrane and pipe bend stressing calculated by FlawPRO from BS 7910 magnification factors for membrane and through-wall bend stressing .................................................10
<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.9</td>
<td>Schematic showing how a reeled pipe can only become straight after unreeling if elastic spring-back is allowed for</td>
</tr>
<tr>
<td>2.10</td>
<td>Comparison of reeling residual stresses calculated using FEA and FlawPRO</td>
</tr>
<tr>
<td>2.11</td>
<td>Example of the effects of shakedown (plastic stress relaxation) on a residual stress field</td>
</tr>
<tr>
<td>3.1</td>
<td>Comparison of FlawPRO SIF solutions for pure bending derived using a 2-D WF with the solution of Benthem and Koiter (1973)</td>
</tr>
<tr>
<td>3.2</td>
<td>Comparison of FlawPRO SIF solutions for a quadratic stress variation derived using a 2-D WF with the solution of Hellen et al. (1982)</td>
</tr>
<tr>
<td>3.3</td>
<td>Comparison of the FlawPRO SIF solutions for semi-circular flaws subjected to uniform stressing and bending with the equivalent solutions calculated using the SC17 crack model in NASGRO</td>
</tr>
<tr>
<td>3.4</td>
<td>Local membrane and through-wall bend stress variations used in the validation of FlawPRO SIF solutions</td>
</tr>
<tr>
<td>3.5</td>
<td>Comparison of M_k factors for nominal membrane stressing calculated by FlawPRO with factors determined using the SC17 crack model in NASGRO</td>
</tr>
<tr>
<td>3.6</td>
<td>Comparison of M_k factors for nominal through-wall bend stressing calculated by FlawPRO with factors determined using the SC17 crack model in NASGRO</td>
</tr>
<tr>
<td>3.7</td>
<td>Illustration of the two offset embedded flaw models in FlawPRO</td>
</tr>
<tr>
<td>3.8</td>
<td>Comparison of FlawPRO (dashed lines) and NASGRO (solid lines) EC01 SIF solutions: central embedded flaw subject to uniform tension</td>
</tr>
<tr>
<td>3.9</td>
<td>Comparison of FlawPRO (dashed lines) and Isida and Noguchi (solid lines) EC03 SIF solutions: embedded flaw approaching free surface (h/2Y=0.5) subjected to uniform tension</td>
</tr>
<tr>
<td>3.10</td>
<td>Comparison of FlawPRO (dashed lines) and Isida and Noguchi (solid lines) EC03 SIF solutions: embedded flaw approaching free surface (h/2Y=0.625) subjected to uniform tension</td>
</tr>
<tr>
<td>3.11</td>
<td>Comparison of FlawPRO (dashed lines) and Isida and Noguchi (solid lines) EC03 SIF solutions: embedded flaw approaching free surface (h/2Y=0.8) subjected to uniform tension</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>3.12</td>
<td>Comparison of FlawPRO EC02 SIF (dashed lines) and Shah and Kobayashi’s exact (solid lines) solutions: embedded flaw in infinite body subjected to stress variations</td>
</tr>
<tr>
<td>3.13</td>
<td>Comparison of FlawPRO EC02 and EC03 SIF solutions: embedded flaw subjected to stress variations approaching a free surface, ( h/2Y=0.85 )</td>
</tr>
<tr>
<td>5.1</td>
<td>Schematic showing the two component J estimation scheme</td>
</tr>
<tr>
<td>5.2</td>
<td>Derivation of reference strain from reference stress</td>
</tr>
<tr>
<td>5.3</td>
<td>Example of flaw embedded in the plastic zone at a notch showing how the hybrid EPRI/reference stress J estimation scheme captures the effects of stress concentration features on the EPFM crack-tip driving force</td>
</tr>
<tr>
<td>5.4</td>
<td>Illustration showing the effects on J of assuming flaws are subjected to load controlled or strain controlled loading during reeling</td>
</tr>
<tr>
<td>5.5</td>
<td>Schematic of how the elastic-plastic stress at the extrados of a pipe during reeling is derived from the reeling strain</td>
</tr>
<tr>
<td>5.6</td>
<td>Typical finite element model used to compute J values for circumferential surface flaws in pipes subjected to reeling</td>
</tr>
<tr>
<td>5.7</td>
<td>Comparison of the two stress-strain curves used in validation of the FlawPRO J estimation scheme for strain controlled reeling</td>
</tr>
<tr>
<td>5.8</td>
<td>Comparison of FEA J results for the deepest points on surface flaws with the predictions of the FlawPRO J estimation scheme for pipes subjected to 1.3% reeling strain</td>
</tr>
<tr>
<td>5.9</td>
<td>Illustration of how the FlawPRO J estimation scheme tended to predict J values for surface points on flaws that were more consistent with the FEA J values for points on the flaws at around 30 degrees from the surface</td>
</tr>
<tr>
<td>5.10</td>
<td>Comparison of FEA J results for the deepest points on surface flaws with the predictions of the FlawPRO J estimation scheme for pipes subjected to 2.6% reeling strain</td>
</tr>
<tr>
<td>5.11</td>
<td>FEA results showing the strain concentration of a reeled pipe subjected to a 3% reeling strain containing a geometric discontinuity due to the welding together of axially misaligned pipes</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>5.12</td>
<td>In order to allow for the strain concentration at the misalignment geometric discontinuity, FlawPRO uses a normalized stress variation that captures the effects of the discontinuity on the elastically determined pipe bend stress.</td>
</tr>
<tr>
<td>6.1</td>
<td>Crack growth rates in steels appear to be R-dependent when plotted against $\Delta K_{\text{total}} (=\Delta K)$ but collapse onto a single curve when plotted against a closure corrected parameter, $\Delta K_{\text{eff}}$.</td>
</tr>
<tr>
<td>6.2</td>
<td>Example of results showing $\Delta J$ correlates fatigue crack growth rates under cyclic linear elastic and cyclic elastic-plastic crack-tip conditions (see N.E. Dowling, 1976).</td>
</tr>
<tr>
<td>6.3</td>
<td>The $dc/dN$ curve is represented in FlawPRO by three regions: Region 1 (below threshold, no growth), and low (Region 2) and high (Region 3) $\Delta K$ growth regions.</td>
</tr>
<tr>
<td>6.4</td>
<td>Schematic showing how the amount of crack-tip blunting and ductile tearing is determined in FlawPRO during reeling.</td>
</tr>
<tr>
<td>6.5</td>
<td>Results of fatigue crack growth rate measurements on small specimens subjected to load control showing the enhancement in growth rates at high $\Delta K_{\text{eff}}$ values due to tear-fatigue.</td>
</tr>
<tr>
<td>6.6</td>
<td>Illustration of flaw growth by tear-fatigue based on the Memory Model.</td>
</tr>
<tr>
<td>6.7</td>
<td>The Paris equation fails to predict crack growth rates under tear-fatigue conditions. The tear-fatigue model incorporated into FlawPRO based on the Memory Model successfully predicts the observed crack growth rates.</td>
</tr>
<tr>
<td>6.8</td>
<td>In agreement with observed behavior, the tear-fatigue model in FlawPRO predicts no acceleration in fatigue rates relative to Paris equation predictions when the applied $J$ versus crack depth curve ($dJ/da$) has zero slope due to a load mode change during testing.</td>
</tr>
<tr>
<td>7.1</td>
<td>Illustration of how the size of a transitional flaw is calculated.</td>
</tr>
</tbody>
</table>
NOMENCLATURE

\( a \quad \text{flaw depth (height) for surface flaws and half the flaw height for embedded flaws} \)

\( a_{e} \quad \text{effective flaw depth including a first order plastic correction} \)

\( a_{\text{new}} \quad \text{flaw depth (height) for surface flaws and half the flaw height for embedded flaws after crack transitioning} \)

\( \frac{da}{dN} \quad \text{fatigue crack growth rate} \)

\( b \quad \text{exponent in power law stress-strain curve} \)

\( c \quad \text{half flaw surface length for surface flaws and half flaw length for embedded flaws} \)

\( c_{\text{new}} \quad \text{half flaw surface length for surface flaws and half flaw length for embedded flaws after crack transitioning} \)

\( A \quad \text{coefficient in power law stress-strain curve} \)

\( C \quad \text{coefficient in Paris equation expressed in terms of } \Delta K \)

\( C' \quad \text{coefficient in Paris equation expressed in terms of } \Delta J \)

\( E \quad \text{Young’s modulus} \)

\( E' = E \text{ for plane stress and } E/(1-\nu^2) \text{ for plane strain situations} \)

\( f(a/t, a/c) \quad \text{SIF surface correction term} \)

\( F_{\text{elastic}} \quad \text{geometric term relating the elastic outer fiber stress to the moment} \)

\( F_{\text{elastic-plastic}} \quad \text{geometric term relating the elastic-plastic outer fiber stress to the moment} \)

\( F_{\text{plastic}} \quad \text{geometric term relating the fully-plastic outer fiber stress to the moment} \)

\( g(a/t, a/c) \quad \text{interpolation function appearing in weight function formulation} \)

\( h \quad \text{flaw height} \)

\( H(a/t, a/c) \quad \text{function appearing in weight function formulation} \)

\( J \quad \text{J-integral} \)

\( J_e \quad \text{elastic component of } J \)

\( J_p \quad \text{plastic component of } J \)
NOMENCLATURE (Continued)

JR \quad J-R\ curve
K \quad \text{stress intensity factor (SIF)}
K^P \quad \text{SIF for primary load}
K^S \quad \text{SIF for secondary load}
K_{2-D}(a/t) \quad \text{SIF for 2-D flaw}
K_{2-D}^0(a/t) \quad \text{SIF for 2-D flaw subjected to uniform stressing}
K_{3-D}(a,c,a/t) \quad \text{SIF at the a-tip of a 3-D flaw}
K_{cir}^a(a/t) \quad \text{SIF at the a-tip of a circular flaw}
K_{3-D}(a,c,a/t) \quad \text{SIF at the c-tip of a 3-D flaw}
K_{cir}^c(a/t) \quad \text{SIF at the c-tip of a circular flaw}
K_{cir}^0(a/t) \quad \text{SIF for circular flaw subjected to uniform stressing}
K_{ref} \quad \text{SIF corresponding to the reference stress used in the weight function formulation}
L \quad \text{length of flaw}
L_{eff} \quad \text{effective flaw length} = \pi L/4
L_{new} \quad \text{length of flaw after crack transitioning}
m \quad \text{exponent in Paris equation}
m(a/t) \quad \text{geometry parameter used in the weight function formulation}
M \quad \text{moment}
M_k(a/t) \quad 2-D \text{ SIF magnification factor defined in BS 7910}
M(a/t,a/c) \quad \text{interpolation function appearing in weight function formulation}
M_{reel} \quad \text{moment applied during reeling}
M_{unreel} \quad \text{unloading moment corresponding to unreeling and to compensate for spring-back during straightening}
NOMENCLATURE (Continued)

\( \text{OD}_{\text{pipe}} \) pipe outer diameter
\( \text{OD}_{\text{spool}} \) spool outer diameter
\( p \) internal pressure
\( P \) applied load
\( P_0 \) net section yield load
\( r \) radial distance
\( r_y \) plastic zone size for small scale yielding
\( R_1 \) inner radius of pipe
\( R_2 \) outer radius of pipe
\( R_m \) mean radius of pipe = \((R_2+R_1)/2\)

\( \text{SCF}_{\text{misalignment}} \) misalignment bend stress concentration factor
\( t \) pipe wall thickness
\( U \) crack closure term
\( V_p \) numerical factor in \( J_p \) estimation scheme
\( x \) distance
\( W(a,x) \) 2-D weight function
\( y \) flaw offset: distance from embedded flaw tip nearest a free surface to the free surface
\( Y \) flaw offset used in validation: distance from center of embedded flaw to nearest free surface: \( Y = y + h/2 \)
\( \alpha \) coefficient in Ramberg-Osgood equation
\( \beta \) plane stress/plane strain factor in first order plastic correction to flaw size
\( \beta_s \) surface correction term appearing in expression for \( \Delta K \)
\( \gamma \) exponent in Ramberg-Osgood equation
\( \Delta a_t \) increment of blunting and ductile tearing
### NOMENCLATURE (Continued)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta K$</td>
<td>cyclic change in the SIF due to load cycling</td>
</tr>
<tr>
<td>$\Delta K_{el-pl}$</td>
<td>plastic corrected $\Delta K$</td>
</tr>
<tr>
<td>$\Delta K_{eff}$</td>
<td>effective crack closure corrected value of $\Delta K$</td>
</tr>
<tr>
<td>$\Delta K_{eff}^{el-pl}$</td>
<td>effective crack closure and plastic corrected $\Delta K$</td>
</tr>
<tr>
<td>$\Delta K_{th}$</td>
<td>fatigue crack growth threshold</td>
</tr>
<tr>
<td>$\Delta K_{trans}$</td>
<td>value of $\Delta K$ corresponding to the demarcation between Region 2 and Region 3 on the $da/dN$ curve</td>
</tr>
<tr>
<td>$\Delta J$</td>
<td>cyclic change in the J due to load cycling</td>
</tr>
<tr>
<td>$\Delta J_e$</td>
<td>cyclic change in elastic component of J due to load cycling</td>
</tr>
<tr>
<td>$\Delta J_{eff}$</td>
<td>effective cyclic change in J corrected for crack closure and surface effects</td>
</tr>
<tr>
<td>$\Delta J_p$</td>
<td>cyclic change in the plastic component of J due to load cycling</td>
</tr>
<tr>
<td>$\Delta r_y$</td>
<td>cyclic change in plastic zone size due to load cycling</td>
</tr>
<tr>
<td>$\Delta \varepsilon$</td>
<td>cyclic change in strain related to cyclic stress-strain curve</td>
</tr>
<tr>
<td>$\Delta \varepsilon^p$</td>
<td>cyclic change in plastic component of reeling strain</td>
</tr>
<tr>
<td>$\Delta \phi_{ref}$</td>
<td>term appearing in $\Delta J_e$</td>
</tr>
<tr>
<td>$\Delta \sigma_{ref}$</td>
<td>cyclic change in reference stress</td>
</tr>
<tr>
<td>$\Delta \sigma$</td>
<td>cyclic change in stress related to cyclic stress-strain curve</td>
</tr>
<tr>
<td>$\Delta \sigma_{ref}$</td>
<td>cyclic change in reference stress</td>
</tr>
<tr>
<td>$\Delta \sigma_{spring}$</td>
<td>spring-back stress induced during straightening</td>
</tr>
<tr>
<td>$\Delta \sigma_y$</td>
<td>cyclic yield stress</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>strain</td>
</tr>
<tr>
<td>$\varepsilon_{elastic}$</td>
<td>elastic strain</td>
</tr>
</tbody>
</table>
### NOMENCLATURE (Continued)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{\text{elastic-plastic}}$</td>
<td>elastic-plastic strain</td>
</tr>
<tr>
<td>$\varepsilon_{\text{ref}}$</td>
<td>reference strain corresponding to reference stress on stress-strain curve</td>
</tr>
<tr>
<td>$\varepsilon^p_{\text{ref}}$</td>
<td>plastic component of reference strain</td>
</tr>
<tr>
<td>$\varepsilon_{\text{reel}}$</td>
<td>maximum reeling strain</td>
</tr>
<tr>
<td>$\varepsilon_o$</td>
<td>Ramberg-Osgood yield strain = $\sigma_o/E$</td>
</tr>
<tr>
<td>$\varepsilon_{\text{stress-strain}}(\sigma)$</td>
<td>the strain on the stress-strain curve corresponding to $\sigma$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>half angle subtended by flaw = $L_{\text{eff}}/2R_m$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$\phi$</td>
<td>term appearing in first order plastic correction to flaw size under load control</td>
</tr>
<tr>
<td>$\phi_{\text{reel}}$</td>
<td>term appearing in first order plastic correction to flaw size under strain controlled reeling</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>stress</td>
</tr>
<tr>
<td>$\sigma(r,\phi)$</td>
<td>stress distribution in pipe</td>
</tr>
<tr>
<td>$\sigma_{\text{elastic}}$</td>
<td>elastic stress</td>
</tr>
<tr>
<td>$\sigma_{\text{membrane}}(x)$</td>
<td>elastic membrane stress</td>
</tr>
<tr>
<td>$\sigma_{\text{elastic-plastic}}$</td>
<td>elastic-plastic stress</td>
</tr>
<tr>
<td>$\sigma_{\text{combined}}_{\text{elastic-plastic}}(x)$</td>
<td>combined membrane and residual stress after shakedown</td>
</tr>
<tr>
<td>$\sigma_{\text{fiber}}$</td>
<td>elastic determined maximum outer fiber stress at extrados of reeled pipe</td>
</tr>
<tr>
<td>$\sigma_{\text{fiber}}(x)$</td>
<td>through-wall change in outer fiber stress</td>
</tr>
<tr>
<td>$\sigma_{\text{flow}}$</td>
<td>elastic-plastic determined maximum outer fiber stress at extrados of reeled pipe</td>
</tr>
<tr>
<td>$\sigma_{\text{membrane}}$</td>
<td>nominal membrane stress</td>
</tr>
<tr>
<td>$\sigma_{\text{normalized}}(x)$</td>
<td>normalized stress variation</td>
</tr>
<tr>
<td>$\sigma^p$</td>
<td>primary stress, such as a membrane or a pipe bend stress, that is induced by applied forces and moments</td>
</tr>
</tbody>
</table>
NOMENCLATURE (Continued)

\( \sigma^S \) secondary stress, such as a residual stress

\( \sigma^{SCF}(x) \) effect on spatial variation of membrane and pipe bend stresses due to weld geometrical discontinuities absent geometrical effects due to axial misalignment

\( \sigma_{\text{normalized membrane}}(x) \) normalized membrane stress variation due to weld geometrical discontinuity

\( \sigma_{\text{net section membrane}} \) membrane stress at net section yielding

\( \sigma_{\text{SCF membrane}}(x) \) spatial variation in membrane stress due to a stress concentration feature

\( \sigma_{\text{misalignment}}(x) \) stress variation due to axial misalignment absent stress concentration features

\( \sigma_{\text{normalized misalignment}}(x) \) normalized stress variation corresponding to the geometrical effects of misalignment on weld geometrical discontinuities

\( \sigma_{\text{SCF misalignment}}(x) \) factor representing the change in spatial variation in pipe membrane and bend stresses due to the geometrical effects of misalignment on weld geometrical discontinuities

\( \sigma_o \) Ramberg-Osgood yield stress

\( \sigma_{\text{pipebend}} \) nominal pipe bend stress

\( \sigma_{\text{normalized pipebend}}(x) \) normalized pipe bend stress variation due to weld geometrical discontinuity

\( \sigma_{\text{net section pipebend}} \) pipe bend stress at net section yielding

\( \sigma_{\text{SCF pipebend}}(x) \) spatial variation in pipe bend stress due to a stress concentration feature

\( \sigma_{\text{residual reel}}(x) \) residual stress due to reeling

\( \sigma_{\text{el-pl reel}}(x) \) elastic-plastic stress at maximum reel strain

\( \sigma_{\text{ref}} \) reference stress related to net section yield load

\( \sigma_{\text{ref}(x)} \) reference stress variation used in the weight function formulation

\( \sigma_{\text{residual shakedown}}(x) \) residual stress after shakedown

\( \sigma_{\text{shakedown}}^S \) secondary stress after shakedown
\( \sigma_{\text{throughbend}} \) nominal through-wall bend stress

\( \sigma_{\text{SCF}}^{x\text{SCF}}(x) \) spatial variation due to the membrane stress, pipe bend stress, weld geometrical discontinuity, axial misalignment, and the geometrical effects of misalignment

\( \sigma_{\text{unreeled}}^{\text{el-pl}}(x) \) elastic-plastic stress change due to unreeling

\( \sigma_y \) yield stress
EXECUTIVE SUMMARY

This report summarizes the main technical aspects that underpin the advanced fracture mechanics technology incorporated in the computer program FlawPRO™ Version 3. FlawPRO is a graphical user interface (GUI) driven program specifically designed for performing engineering critical assessments of reeled and conventionally installed sub-sea pipes.

The technical issues covered in this report include:

- linear elastic and elastic-plastic stress analysis related to reeling, plastic relaxation of residual stresses, and through-wall pipe stress variations resulting from weld geometrical discontinuities and axial mismatch;
- determination and validation of the weight function approach adopted in FlawPRO for calculating the stress intensity factors of offset embedded and surface flaws subjected to spatial stress variations;
- calculation of net section yield loads for offset embedded, surface and through-wall flaws subjected to combined axial forces and moments;
- estimation and validation of the schemes included in FlawPRO for determining the elastic-plastic fracture mechanics (EPFM) parameters \( J \) and \( \Delta J \) under strain controlled (as pertains during reeling) and load controlled (as pertains under worst-case loading conditions that could cause failure during installation fatigue and service fatigue) conditions;
- mechanics of flaw extension due to tear-fatigue (i.e. combined ductile tearing and low cycle fatigue) during reeling and fatigue crack growth during installation and service;
- crack transitioning of embedded flaws into surface flaws and surface flaws into through-wall flaws during crack growth.

Much of the advanced methodology incorporated in FlawPRO for assessing flaw growth during reeling is not included in standard engineering critical assessment procedures such as BS 7910:1999. In particular, these procedures give no guidance on the determination of the parameters \( J \) and \( \Delta J \) under strain controlled loading and the mechanics of treating tear-fatigue, especially under low cycle fatigue conditions. It is assumed in this report that tear-fatigue is the mechanism controlling flaw extension during reeling and this assumption is validated against full-scale tests of flawed pipes. Both the evaluation of \( J \) and \( \Delta J \) and the treatment of combined flaw extension by ductile tearing and low cycle fatigue are technical issues crucial to an accurate and physically meaningful assessment of flaw extension during reeling.
1.0 INTRODUCTION

FlawPRO™ Version 3 is a computer program for performing engineering critical assessments (ECAs) of reeled and unreeled sub-sea pipe. It incorporates stress analysis procedures and advanced elastic-plastic fracture mechanics (EPFM) concepts to determine the effects of reeling on crack extension from pre-existing girth weld flaws.

Reeling subjects a pipe to high strains, typically around 2%, that plastically deforms all or most of the pipe. A single reel involves two bending and straightening procedures. Pipe sections are first welded together onshore and the welded pipe string is consequently bent around a spool and transported by ship to the location where it is to be installed. During installation, the string is pulled off the spool and straightened before being bent around an aligner and finally straightened again. This procedure subjects the pipe to monotonic and cyclic straining. The monotonic straining can produce crack extension from pre-existing flaws by ductile tearing. The cyclic straining can cause additional crack extension due to fully reversed low cycle fatigue (LCF).

The ECA for reeled pipe performed in FlawPRO takes into account the simultaneous ductile tearing and LCF by using a tear-fatigue model that captures the synergy between these two mechanisms of flaw growth. The model requires the evaluation of EPFM parameters, such as $J$ and $\Delta J$, that represent crack-tip driving forces under monotonic and cyclic loading conditions, respectively. Since reeling occurs under constant strain conditions (determined by the diameter of the pipe and the spool), the parameters $J$ and $\Delta J$ are evaluated in FlawPRO for strain controlled loading.

A pipe may experience cyclic loading after reeling but before entering service, such as when the pipe string hangs overboard and is subjected to wave induced loads. This installation fatigue, which can cause further pre-service crack extension to that already produced by reeling, generally involves elastic straining. FlawPRO employs conventional linear elastic fracture mechanics (LEFM) based on the stress intensity factor (SIF), $K$, and the cyclic change in the SIF, $\Delta K$, in order to calculate crack growth from installation fatigue. Similarly, after the pipe has entered service, FlawPRO employs $K$ and $\Delta K$ when calculating fatigue crack growth under service loading conditions. Account is taken of the possibility of failure from unlikely but severe monotonic loads (worst case stressing) during installation fatigue and service fatigue using a load controlled formulation of the EPFM parameter $J$.

The calculation of the EPFM crack-tip driving forces $J$ and $\Delta J$ require knowledge of the applied loads (expressed in terms of the magnitude and variation of the stresses local to a postulated flaw site), material stress-strain behavior, SIF solutions, and net section yield loads. The evaluation of flaw growth requires mechanistic models for tear-fatigue, fatigue crack propagation in the threshold and so-called Paris regimes, and procedures for allowing propagating cracks to transition when they intersect a pipe surface (inner or outer), such as occurs when an embedded flaw grows through to a surface and becomes a surface flaw.

This theoretical report describes aspects of the foregoing parameters, models, material properties, and procedures that are used in FlawPRO. In some cases (e.g. for SIF solutions),
validation is presented in support of the adopted approach. The report is divided into six technical sections and four other sections: this introduction, conclusions, acknowledgements, and a list of references.

The contents of each of the six technical sections are briefly described below:

**Section 2.0: Stress Analysis:** This section provides relationships between applied axial loads, internal pressure, and moments and the axial stress in the wall of a pipe. It is shown how the axial pipe stresses are influenced by local stress concentrations arising from weld geometrical discontinuities and the geometrical manifestation of pipe misalignment. In FlawPRO, the changes in axial stresses due to stress concentrators are allowed for via normalized stress variations that multiply the nominal axial stresses. These normalized variations can be specified by the user in tabular form, or automatically estimated from SIF magnification factors provided in BS 7910. Examples of the latter procedure are given. Aspects of elastic-plastic stress analysis are also presented in this section. These relate to constant strain based analyses that are applicable to reeling in order to determine the elastic spring-back stress that arises during straightening and the residual stress that remains in the pipe after the completion of the reeling operation. Elastic-plastic issues related to plastic stress relaxation of residual stresses when combined with primary loads are also described.

**Section 3.0: Stress Intensity Factors:** The weight function (WF) method is used in FlawPRO to calculate SIFs for embedded and surface flaws subjected to spatial variations in stress. This section briefly describes the WF method and the formulations used to evaluate these functions. Validation of the approach is provided by comparing the FlawPRO SIF solutions for offset embedded and surface flaws with solutions calculated using other software packages (such as NAGRO) and solutions published in the open literature. Examples of SIF magnification factors for surface flaws emanating from weld geometrical discontinuities are given.

**Section 4.0: Net Section Yield Loads and Reference Stresses:** Expressions for the axial loads (forces and moments) needed to cause net section yielding of a flawed section of pipe containing either an embedded, a surface, or a through-wall crack are given in this section. These expressions are redefined in terms of the reference stresses for combined axial forces and moments that are used in the J formulation methods described in Section 5.

**Section 5.0: J Estimation Schemes for Reeling, Installation and Service:** A brief description of the J and ΔJ formulation methods for strain controlled and load controlled conditions is given in this section. Expressions for J and ΔJ are presented, and the treatment of residual stresses is illustrated. Validation of the J estimation methods is provided for surface flaws at stress concentrators subjected to load control, and for surface flaws subjected to simulated strain controlled reeling. The way the strain controlled J formulation is modified to derive expressions for ΔJ and the application of this parameter to predicting flaw growth under the LCF conditions pertaining during reeling is described.

**Section 6.0: Mechanics of Crack Growth:** General principles behind fatigue crack growth in the Paris regime are described in this section together with how these concepts are extended to the LCF conditions encountered during reeling where cyclic crack extension is
described by $\Delta J$ rather than $\Delta K$. Justification for the use of $\Delta J$ rather than $\Delta K$ under cyclic plastic conditions is presented. The approach adopted in FlawPRO for treating flaw growth in the threshold and intermediate growth rate regimes based on the Paris equation are described. The methodologies for evaluating ductile tearing and tear-fatigue during reeling are reviewed. Evidence supporting the tear-fatigue model employed in FlawPRO to calculate flaw extension during reeling is presented.

Section 7.0: Crack Transitioning: In this section the FlawPRO approach to crack transitioning is described. The conditions for crack transitioning are listed. The types of transitions that are permissible and how flaw sizes are estimated after transitioning are specified.

This theoretical report does not address the use of the results of full scale simulated reeling of flawed welded pipes in the validation of the reeling methodology in FlawPRO. The successful validation of FlawPRO reeling predictions against full scale test results is reported in the Final Report for the Joint Industry Project titled *Validation of a Methodology for Assessing Defect Tolerance of Welded Reeled Risers* performed by Southwest Research Institute (SwRI®).
2.0 STRESS ANALYSIS

The loads acting on a welded pipe are illustrated in Figure 2.1. The loads consist of axial forces (which give rise to a nominal membrane stresses) and moments (which give rise to nominal pipe bend stresses). The applied moments include the moment that is induced by axial forces due to axial misalignment of two welded pipes.

The nominal stresses due to membrane and pipe bend stresses may be locally enhanced by weld geometrical discontinuities that occur at the weld cap and weld root, for example, due to the presence of a weld toe.

![Figure 2.1: Schematic of the loads acting on welded pipes.](image)

The local stress variations through the pipe wall are required in FlawPRO to calculate the SIFs for surface and embedded flaws using the appropriate weight functions for these flaw types. Only nominal stresses due to membrane and pipe bending are needed to evaluate through-wall flaws.

2.1 Elastic Analysis

2.1.1 General

In FlawPRO, it is assumed that the applied loads on a pipe can be expressed as a combination of membrane, pipe bend and residual stresses. The axial membrane stress can be derived from an axial load using the equation:
\[ \sigma_{\text{membrane}} = \frac{P}{\pi (R_2^2 - R_1^2)} \]  

(2.1)

In the case of internal pressure, \( p \), Equation (2.1) is still applicable but with the axial force \( P \) replaced by \( \pi R_2^2 p \).

The axial pipe bend stress can be derived from an applied moment using the expression:

\[ \sigma_{\text{pipebend}} = \frac{4MR_2}{\pi (R_2^4 - R_1^4)} \]  

(2.2)

Absent stress concentration features such as weld geometrical discontinuities the membrane stress is uniformly distributed through the pipe wall whereas the pipe bend stress has a linear variation of the form:

\[ \sigma(x) = \sigma_{\text{pipebend}} \left( 1 - \frac{2x}{OD} \right) \]  

(2.3)

Absent weld geometrical discontinuities, the membrane and pipe bend stresses given in these equations represent local nominal stresses. If a stress concentration is present in a pipe weld then the local nominal stresses are factored by normalized stress variations that capture the effects of the stress concentration factor as shown in the following equations:

\[ \sigma_{\text{SCF membrane}}^{\text{normalized}}(x) = \sigma_{\text{membrane}}^{\text{SCF}} \sigma_{\text{membrane}}(x) \]  

(2.4)

\[ \sigma_{\text{SCF pipebend}}^{\text{normalized}}(x) = \sigma_{\text{pipebend}}^{\text{SCF}} \sigma_{\text{pipebend}}(x) \]  

(2.5)

If the axes of two welded pipes are misaligned, then the axial load corresponding to a membrane stress will induce an additional pipe bend stress of the form:

\[ \sigma_{\text{misalignment}}(x) = (SCF_{\text{misalignment}} - 1) \sigma_{\text{membrane}} \left( 1 - \frac{2x}{OD} \right) \]  

(2.6)

Axial misalignment may also change the stress concentration effects caused by weld geometrical discontinuities that are assumed in aligned pipe, as illustrated in Figure 2.2. These changes are in addition to the pipe bend stress induced by membrane stressing when misalignment is present. To allow for this perturbation in the weld discontinuity, a normalized stress variation is introduced in FlawPRO defined through the equation:

\[ \sigma_{\text{SCF misalignment}}^{\text{normalized}}(x) = \frac{\sigma_{\text{SCF misalignment}}(x)}{\sigma_{\text{SCF}}(x)} \]  

(2.7)
The total stress variation due to a membrane stress, pipe bend stress, weld geometrical discontinuity, misalignment and the geometrical effects of misalignment is given by:

\[
\sigma_{\text{total}}^{SCF}(x) = \left[\sigma_{\text{membrane}}^{\text{normalized}}(x) + \sigma_{\text{pipebend}}^{\text{normalized}}(x) + \sigma_{\text{membrane}}^{\text{SCF_misalignment}} - 1\right] \sigma_{\text{normalized_misalignment}}(x)
\]

(2.8)

Figure 2.2: Illustration of how misalignment may result in geometrical changes to weld geometrical discontinuities assumed for aligned pipe.

2.1.2 Determining Normalized Stress Variations Using BS 7910

Whereas in FlawPRO the normalized stress variation due to the geometrical effects of misalignment has to be defined by the user, the normalized stress variations due to the effects of weld discontinuities on membrane and pipe bend stresses can either be user defined or calculated by the program, as shown below. The approach described below uses the stress intensity factor (SIF) magnification factors for 2-D flaws at weld discontinuities subjected to remote membrane and through-wall bend stresses given in section M.5.1.2 in Annex M of BS 7910. Note that the pipe bend stress is related to the through-wall bend stress \(\sigma_{\text{throughbend}}^{\text{BS7910}}\) and the membrane stress as they are defined in BS 7910, \(\sigma_{\text{membrane}}^{\text{BS7910}}\) by the equation:

\[
\sigma_{\text{pipebend}}(x) = \sigma_{\text{membrane}}^{\text{BS7910}} + \sigma_{\text{throughbend}}^{\text{BS7910}} \left[1 - \frac{2x}{t}\right] = \left(1 - \frac{t}{OD}\right) \sigma_{\text{pipebend}} + \frac{t}{OD} \sigma_{\text{pipebend}} \left(1 - \frac{2x}{t}\right)
\]

(2.9)
The BS 7910 SIF magnification factors for weld discontinuities are defined by:

\[ M_k = \frac{K(\text{evaluated with weld SCF})}{K(\text{evaluated with no weld})} \] (2.10)

The SIF can be evaluated for a 2-D flaw subjected to a stress variation using the weight function method. Thus,

\[ K(\text{evaluated with weld SCF}) = \int_{0}^{a} W(a,x)\sigma^{SCF}(x)dx \] (2.11)

\[ K(\text{evaluated with no weld}) = \int_{0}^{a} W(a,x)\sigma(x)dx \] (2.12)

and

\[ M_k(a/t) = \frac{\int_{0}^{a} W(a,x)\sigma^{SCF}(x)dx}{\int_{0}^{a} W(a,x)\sigma(x)dx} \] (2.13)

Provided the magnification factor, \( M_k(a/t) \), the nominal stress, \( \sigma(x) \), and the weight function, \( W(a,x) \), are known, Equation (2.13) can be solved for the stress variation \( \sigma^{SCF}(x) \). This is the approach adopted in FlawPRO which contains a weight function for a 2-D flaw. The normalized stress variation is then given by:

\[ \sigma_{\text{normalized}}(x) = \frac{\sigma^{SCF}(x)}{\sigma(x=0)} \] (2.14)

where \( \sigma(x=0) \) represents either the nominal membrane stress, the nominal through-wall bend stress, or the nominal pipe bend stress.

Typical normalized stress variations for membrane and pipe bend stresses are shown in Figure 2.3 together with the \( M_k \) factors they were derived from. The discontinuous behavior in the derived stresses with \( x/t \) is due to the discontinuous behavior in the values of the \( M_k \) factors given in BS 7910. A comparison is given in Figure 2.4 between BS 7910 magnification factors for membrane and through-wall bend stressing and similar factors calculated using the weight function method in FlawPRO and the normalized stresses estimated by FlawPRO procedures based on the BS 7910 \( M_k \) factors shown in Figure 2.3. The excellent agreement between the BS 7910 and FlawPRO \( M_k \) factors validates the FlawPRO procedures used to derive the normalized stresses.

FlawPRO predictions for the \( M_k \) factors for deepest and surface points on semi-elliptical surface flaws at weld geometrical discontinuities subjected to nominal membrane and through-wall bend stressing are shown in Figures 2.5 and 2.6, respectively. These results illustrate that while the \( M_k \) factors at the deepest points on surface flaws fall rapidly with increasing flaw depth
in a manner similar to the $M_k$ factors for 2-D flaws, the $M_k$ factors for the surface points remain relatively high and fall less rapidly since the surface points always experience the high stresses at the roots of the geometrical discontinuities. The singularity in the FlawPRO $M_k$ factor for a semi-circular flaw subjected to through-wall bending is a consequence of the definition of the $M_k$ factors as the ratio of the SIFs for flaws at welds with stress concentrating features (SCF) to the SIFs for flaws absent the weld discontinuities, as demonstrated by the results presented in Figure 2.7.

Figure 2.3: Comparison of the BS 7910 magnification factors $M_k$ for 2-D flaws at weld geometrical discontinuities subjected to membrane and through-wall bend stressing with the variation in local stresses derived from these factors using the procedures in FlawPRO.

Figure 2.4: Demonstration of the consistency of the FlawPRO calculated $M_k$ factors for flaws at weld geometrical discontinuities subjected to nominal membrane and through-wall bend stressing (red curve) with the BS 7910 magnification factors (blue curve).
Figure 2.5: Typical $M_k$ factors for 3-D flaws at weld geometrical discontinuities subjected to nominal membrane stressing calculated using the weight function SIF routines in FlawPRO.

Figure 2.6: Typical $M_k$ factors for 3-D flaws at weld geometrical discontinuities subjected to nominal through-wall bend stressing calculated using the weight function SIF routines in FlawPRO.

Figure 2.7: Illustration of the cause of the singularity in the FlawPRO derived $M_k$ factor for deep semi-circular flaws subjected to through-wall bending. The SIFs for semi-circular flaws become small and eventually zero so the ratio of SIF solutions with and without the weld geometrical discontinuity becomes infinite.
Although normalized stress variations displaying discontinuous behaviors such as those shown in Figure 2.3 are calculated by FlawPRO these are not directly used in the program. Instead a “smoothing” procedure is applied to the calculated stressing and the smoothed stress variations are used. Examples of smoothed stress variations for nominal membrane and pipe bend stressing are shown in Figure 2.8. As shown by Equation 2.9, the stress variation for pipe bend stressing can be derived from the solutions for membrane and through-wall bend stressing.

![Normalized Stress Variations](image1)

![Normalized Stress Variations](image2)

**Figure 2.8:** Examples of typical “smoothed” normalized local stress variations for nominal membrane and pipe bend stressing calculated by FlawPRO from BS 7910 magnification factors for membrane and through-wall bend stressing.

### 2.2 Elastic-Plastic Analysis

Two types of elastic-plastic stress analysis are performed in FlawPRO. The first pertains to reeling conditions where straining occurs under constant strain dictated by the curvatures of the spool and aligner. Analyses under these conditions is performed to derive the maximum outer fiber stress at the extrados of the pipe during reeling, and the residual stress that remains in a reeled pipe after final straightening.

The second type of elastic-plastic stress analysis is appropriate for installation fatigue and service fatigue conditions where plastic relaxation of stresses occurs under load controlled conditions. In FlawPRO, the Neuber equation is used in these cases to convert linear elastic derived stress analysis results into an approximate elastic-plastic solution. Analyses under these conditions are used to derive the form of plastically relaxed residual stresses (either due to reeling or welding) after elastic shakedown has occurred because the yield stress is exceeded due to the combined stressing from primary loads and residual stresses (see Equation (2.27) and Section 2.2.3 for more details). Absent residual stresses, pipes under normal operating conditions will usually be designed so that the combined applied membrane and pipe bend stresses are well below yield. However, geometrical discontinuities at welds can cause localized stress concentrations that result in local yielding. FlawPRO calculates the plastically relaxed stresses at these concentration features and estimates the residual stresses that result from the residual plastic strains due to yielding. These localized residual stresses are determined and assumed present in the pipe when reeling and welding residual stresses are absent.
2.2.1 General

Reeling subjects a pipe to a linear strain variation across the diameter of the form:

\[ \varepsilon(x) = \varepsilon_{reel} \left( 1 - \frac{2x}{OD} \right) \]  

(2.15)

The stress variation at \( x \) corresponding to this strain is determined from the stress-strain curve as the root of the equation:

\[ \sigma(x) = \varepsilon_{\text{stress-strain}}(\sigma(x)) \]  

(2.16)

The applied moment corresponding to the reeling strain is obtained from the equation:

\[ M = 2 \int_0^{\frac{\pi}{2}} d\phi \int_{R_2}^{R_1} r dr \sigma(r, \phi) r \cos \phi \]  

(2.17)

This can be integrated for linear elastic behavior to give:

\[ M = \frac{\pi \sigma_{\text{fiber}}}{4} \frac{(R_2^4 - R_1^4)}{R_2} = \sigma_{\text{fiber}} F_{\text{elastic}}(R_1, R_2) \]  

(2.18)

In a fully yielded pipe the moment is:

\[ M = \frac{4\sigma_y}{3} (R_2^3 - R_1^3) = \sigma_y F_{\text{plastic}}(R_1, R_2) \]  

(2.19)

The elastically determined outer fiber stress corresponding to this yield moment is:

\[ \sigma_{\text{fiber}} = \sigma_y \frac{16R_2(R_2^3 - R_1^3)}{3\pi(R_2^4 - R_1^4)} = \sigma_y \frac{F_{\text{plastic}}(R_1, R_2)}{F_{\text{elastic}}(R_1, R_2)} \]  

(2.20)

In general, for an elastic-plastic material,

\[ M = \sigma_{\text{flow}} F_{\text{elastic-plastic}}(R_1, R_2) \]  

(2.21)

where the flow stress \( \sigma_{\text{flow}} \) is the maximum outer fiber stress corresponding to the reel strain and the function \( F_{\text{elastic-plastic}}(R_1, R_2) \) has to be determined by numerically integrating Equation (2.17). The equivalent to Equation (2.20) becomes:

\[ \sigma_{\text{fiber}} = \sigma_{\text{flow}} \frac{F_{\text{elastic-plastic}}(R_1, R_2)}{F_{\text{elastic}}(R_1, R_2)} \]  

(2.22)
In FlawPRO, Equation (2.22) is used to determine $\sigma_{\text{fiber}}$ which is used in the evaluation of $J$ under strain-controlled reeling conditions and a similar equation (involving $\Delta \sigma_{\text{fiber}}$) is used to determine $\Delta J$.

### 2.2.2 Constant Strain-based Stress Analysis Related to Reeling

**Determining the Spring-back Strain During Straightening After Unreeling**

After unreeling from the spool a reeled pipe will be straight only if there are no residual strains present due to reeling. To guarantee this, the pipe has to be subjected to a negative outer fiber strain to compensate for the elastic strain recovered during unloading, as shown in Figure 2.9. The additional increment of strain needed to leave the pipe straight while unloaded after unreeling is herein called the spring-back strain. As can be seen from Figure 2.9, although the pipe is strain free after straightening, it is still stressed due to the presence of reeling residual stresses.

The reversed strain that has to be applied to a reeled pipe from unloading and spring-back during unreeling in order to leave it straight is given by:

$$\varepsilon_{\text{unreel}}(x) = \varepsilon_{\text{reel}}[1-x/R_2] + \frac{\Delta \sigma_{\text{spring}}[1-x/R_2]}{E}$$  \hspace{1cm} (2.23)

In this equation, the spring-back strain $\frac{\Delta \sigma_{\text{spring}}}{E}$ is unknown but is determined in FlawPRO as follows. First, the moment applied during reeling, $M_{\text{reel}}$, is calculated by evaluating the elastic-plastic stress variation in the pipe at the end of reeling, $\sigma_{\text{reel}}^{\text{el-pl}}(x)$, using Equations (2.15), (2.16) and (2.17) combined with monotonic stress-strain properties. Second, the elastic-plastic stress, $\sigma_{\text{unreel}}^{\text{el-pl}}(x)$, corresponding to the strain distribution given in Equation (2.23) is evaluated using Equation (2.16), cyclic stress-strain properties, and a guess for the spring-back stress term in Equation (2.23). Third, the change in moment (unreel moment) $M_{\text{unreel}}$ is evaluated from $\sigma_{\text{unreel}}^{\text{el-pl}}(x)$ using Equation (2.17). Fourth, the spring-back stress is estimated from the equation:

$$\Delta \sigma_{\text{spring}} = \frac{M_{\text{unreel}} - M_{\text{reel}}}{F_{\text{elastic}}(R_1,R_2)}$$  \hspace{1cm} (2.24)

The foregoing four steps are then repeated using $\Delta \sigma_{\text{spring}}$ as the new guess for the spring-back stress until the new guess equals the previous one.
**Determining the Reeling Residual Stress**

The residual stress in the pipe after unreeling and straightening is given by:

$$\sigma_{\text{residual}}(x) = \sigma_{\text{reel}}^{\text{pl}}(x) - \sigma_{\text{unreel}}^{\text{pl}}(x) + \Delta \sigma_{\text{spring}} \left(1 - \frac{x}{R_2}\right) \quad (2.25)$$

The residual stresses are generated in the first deformation cycle of the reeling process when the pipe is straightened after reeling off the spool. These residual stresses are wiped out during the second deformation cycle as the pipe passes over the aligner but are then regenerated during the straightening process following alignment as the deformation loop is completed (see Figure 2.9).

The approach to calculating the residual stress in FlawPRO has been validated against the results of FEA performed by SwRI, as shown in Figure 2.10. This figure illustrates the residual stress variation through the wall of a pipe measured from the OD at the extrados (where maximum straining occurs) after applying a reeling strain of 2% followed by unreeling and straightening.

![Figure 2.9: Schematic showing how a reeled pipe can only become straight after unreeling if elastic spring-back is allowed for.](image-url)
2.2.3 Constant Load-based Stress Analysis Related to Installation and Service

*General*

Applied primary membrane and pipe bend stresses together with welding or reeling residual stresses and stress concentration effects at weld geometrical discontinuities can cause local yielding and plastic stress relaxation. This relaxation, herein called shakedown, can significantly reduce residual stresses and/or primary stresses.

FlawPRO performs a shakedown analysis by applying the Neuber equation. This analysis is done predominantly to determine the variation in residual stress in the wall of a pipe after plastic relaxation. The J estimation formulation employed in FlawPRO uses the linear elastic stress variation corresponding to the primary (membrane and pipe bend) stress not the plastically relaxed variation. The influence of plastic relaxation on J is assumed to manifest itself only through the change in the residual stress due to shakedown.

Using subscripts to represent elastic and elastic-plastic quantities, respectively, then the Neuber equation has the form:

\[
\sigma_{\text{elastic}} \varepsilon_{\text{elastic}} = \sigma_{\text{elastic-plastic}} \varepsilon_{\text{elastic-plastic}} (\sigma_{\text{elastic-plastic}})
\]  

(2.26)
The elastic-plastic stress and strain are related through the stress-strain curve (see Equation (2.16)), enabling $\sigma_{\text{elastic-plastic}}$ to be solved as the root of Equation (2.26).

**Determining the Relaxed Residual Stress after Shakedown**

In FlawPRO, the membrane and pipe bend stresses are combined with the axial misalignment stress factor and the normalized stress variation due to the weld geometrical discontinuity to obtain the through-wall stress variation due to all primary stresses. A typical through-wall stress variation resulting from membrane stressing is shown in Figure 2.11 (top figure, red curve). Also shown in this figure is a residual stress that results from reeling (green dash-dot curve) together with the combined stress (blue dashed curve). Since the weld material yield stress is 690 MPa, it is clear that plastic relaxation of the combined stresses will occur in a local region around the weld stress concentration and result in shakedown.

In FlawPRO, the Neuber equation is applied to each stress point through the pipe wall to estimate the effects of yielding. The results of doing this are shown in the lower figure in Figure 2.11 which displays the combined stress variation after plastic stress relaxation (blue dashed curve). This combined stress consists of the unaltered membrane stress (red curve) plus the residual stress (green dash-dot curve) changed by shakedown. The latter is given by:

$$\sigma_{\text{residual, shakedown}}(x) = \sigma_{\text{combined}}(x) - \sigma_{\text{membrane}}(x)$$

(2.27)
Figure 2.11: Example of the effects of shakedown (plastic stress relaxation) on a residual stress field.
### 3.0 STRESS INTENSITY FACTORS

The stress intensity factor (SIF), $K$, is a crack-tip driving force under linear elastic conditions. It is a key parameter in the fracture mechanics methods used in FlawPRO and contributes to both the elastic and plastic components of the elastic-plastic crack-tip driving force parameter, $J$, which plays a major role in the reeling methodology incorporated in FlawPRO. Similarly, the cyclic change in the stress intensity factor, $\Delta K$, is a key parameter in determining the cyclic crack-tip driving forces used in FlawPRO to calculate fatigue crack growth rates under installation and service conditions.

In FlawPRO, $K$ and $\Delta K$ are evaluated for surface and embedded flaws from stresses in pipes using the weight function method. This technique enables stress intensity factors for flaws in steep stress gradients such as those associated with weld geometrical discontinuities to be calculated based on stress analysis results for the flaw-free pipe.

The ranges of applicability of the FlawPRO SIF solutions in terms of OD, $t$, $h$, $L$, and $y$ are shown in Table 3.1.

**Table 3.1: Ranges of Applicability of FlawPRO SIF Solutions.**

<table>
<thead>
<tr>
<th>Flaw Type</th>
<th>(OD/$t$) Range</th>
<th>(h/$t$) or (h/$y$) Range</th>
<th>(h/$L$) Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Embedded Flaw Offset $y$ from Outside of Pipe</td>
<td>No Limitation (Solution Based on Embedded Flaw in Plate)</td>
<td>$\max\left(\frac{h}{h + 2y}, \frac{h}{2t - 2y - h}\right) \leq 0.9$</td>
<td>$\frac{h}{L} \leq 2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\frac{h}{L} = 2$ solution used when $\frac{h}{L} &gt; 2$</td>
</tr>
<tr>
<td>Embedded Flaw Offset $y$ from Inside of Pipe</td>
<td>No Limitation (Solution Based on Embedded Flaw in Plate)</td>
<td>$\max\left(\frac{h}{h + 2y}, \frac{h}{2t - 2y - h}\right) \leq 0.9$</td>
<td>$\frac{h}{L} \leq 2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\frac{h}{L} = 2$ solution used when $\frac{h}{L} &gt; 2$</td>
</tr>
<tr>
<td>Surface Flaw on Outside of Pipe</td>
<td>$\frac{OD}{t} \leq 200$</td>
<td>$\frac{h}{t} \leq 0.9$</td>
<td>$\frac{h}{L} \leq 1$</td>
</tr>
<tr>
<td></td>
<td>$\left(\frac{OD}{t}\right) = 200$ solution used when $\left(\frac{OD}{t}\right) &gt; 200$</td>
<td></td>
<td>$\frac{h}{L} = 1$ solution used when $\frac{h}{L} &gt; 1$</td>
</tr>
</tbody>
</table>
### 3.1 Weight Function Method

#### 3.1.1 General

The weight function (WF) represents the stress intensity factor solution for two unit line loads, $P$, applied symmetrically to the top and bottom surfaces of a crack. The WF depends on geometry (crack shape and size, position on the crack front, and the type of structure) and the boundaries of the structure over which displacements are prescribed. It is not a function of the form of the applied loading (e.g., whether the loads are due to internal pressure or applied moments) and does not depend on the origins of the applied stresses (e.g., whether they arise from mechanical loads, internal pressure, or residual strains). Furthermore, the stress variation used in the integral over the crack surface performed to determine the SIF using the WF approach is the stress variation derived for the flaw-free structure. These attributes (that the WF is only a geometrical term independent of the source of loading and requires only the results of a flaw-free structural stress analysis) make the WF approach a very powerful and versatile method for calculating SIFs.

The SIFs for flaws with two degrees of freedom (2-DOF), such as surface semi-elliptical flaws, are calculated using equations of the form:

\[
K_u(a,c) = \int_0^a W_u(a,c,x)\sigma(x)dx \tag{3.1}
\]

\[
K_c(a,c) = \int_0^a W_c(a,c,x)\sigma(x)dx \tag{3.2}
\]

where $a$ and $c$ are the flaw depth (height) and half the surface length, respectively, and subscripts $u$ and $c$ signify the deepest and surface points on the flaw, respectively. The term $W(a,c,x)$

---

**Surface Flaw on Inside of Pipe**

<table>
<thead>
<tr>
<th>Condition</th>
<th>Equation</th>
<th>Solution</th>
<th>Used When</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8 \leq \frac{OD}{t} \leq 200$</td>
<td>$\frac{h}{t} \leq 0.9$</td>
<td>$\frac{h}{L} \leq 1$</td>
<td>$\frac{h}{L} = 1$ solution used when $\frac{h}{L} &gt; 1$</td>
</tr>
<tr>
<td>($\frac{OD}{t} = 8$ solution used when $\frac{OD}{t} &lt; 8$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{OD}{t} &gt; 200$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Through Wall Flaw**

<table>
<thead>
<tr>
<th>Condition</th>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Limitation</td>
<td>$\frac{h}{\pi OD} \leq 0.9$</td>
<td>Not Applicable</td>
</tr>
</tbody>
</table>

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represents the WF and \( \sigma(x) \) represents the applied stress variation determined from an analysis of the flaw-free pipe.

Similar equations apply to calculating the SIFs for embedded flaws offset distance \( y \) from the inside or outside surfaces of a pipe. These equations have the form:

\[
K_{a+}(a, c, y) = \int_{-a}^{a} W_{a+}(a, c, y, x) \sigma(x) dx
\]  
(3.3)

\[
K_{c}(a, c, y) = \int_{-a}^{a} W_{c}(a, c, y, x) \sigma(x) dx
\]  
(3.4)

\[
K_{a-}(a, c, y) = \int_{-a}^{a} W_{a-}(a, c, y, x) \sigma(x) dx
\]  
(3.5)

where \( - \) indicates the point on the flaw nearest the free surface from which the flaw is offset, \( + \) indicates the point furthest from this free surface, and \( c \) are the points on the tip of the flaw midway between \( a+ \) and \( a- \).

### 3.1.2 Weight Function Formulation

The weight function for 2-D flaws can be approximately expressed in the form:

\[
W(a, x) = \frac{2}{\pi} \frac{a^{1/2}}{(a^2 - x^2)^{1/2}} [1 + m(a/t) \left( 1 - \frac{x}{a} \right)^{1/2}]
\]  
(3.6)

where \( m(a/t) \) is a function that has to be determined. This is accomplished using a known SIF solution (the reference K solution) corresponding to a known stress (the reference stress variation) and writing:

\[
K_{ref}(a) = \int_{0}^{a} \sigma_{ref}(x) W(a, x) dx
\]  
(3.7)

The unknown function, \( m(a/t) \) can be determined from this equation.

In order to evaluate the SIF for a 3-D surface flaw subjected to an arbitrary stress variation in FlawPRO, the 2-D WF is combined with the SIF solution for a circular flaw. The SIF solutions for the deepest (a-tip) and surface (c-tip) points on the flaw are then calculated using the following two expressions:

\[
K_{s-d}^a(a, c, a/t) = \frac{M(a/t, a/c) \sqrt{\pi a}}{E(k)} \left[ (1 - F(a/c)) \frac{K_{2-D}^a(a/t)}{K_{2-D}^a(a/t)} + F(a/c) \frac{K_{cir}^a(a/t)}{K_{cir}^a(a/t = 0)} \right]
\]  
(3.8)
\[ K_{I,d} (a,c,a/t) = \frac{M(a/c,a/t) \sqrt{\pi a}}{E(k)} f(a/t,a/c) \frac{K^c_{cr}(a/t)}{K^c_{cr}(a/t = 0)} \]  

where the terms appearing in the expressions are defined as follows:

\[ F(a/c) = \left( \frac{a}{c} \right) \left[ 2.402 - 2.911 \left( \frac{a}{c} \right) + 1.507 \left( \frac{a}{c} \right)^2 \right] \]  

\[ f(a/t,a/c) = 1.1 + 0.078 \sqrt{\frac{a}{c}} + [0.12 + 0.23(1 - \frac{a}{c})^2] \left( \frac{a}{t} \right)^2 \]  

\[ M(a/t,a/c) = (1.12 - 0.08 \frac{a}{c})[(1 - g(a/t,a/c)) \frac{K^0_{2-D}(a/t)}{K^0_{2-D}(a/t = 0)} + g(a/t,a/c) \frac{K^0_{cr}(a/t)}{K^0_{cr}(a/t = 0)}] \]  

\[ g(a/t,a/c) = \left[ \frac{3.491 \left( \frac{a}{t} \right)^2 + 11.57 \left( \frac{a}{t} \right)^4}{3.297 \left( \frac{a}{t} \right)^2 + 11.67 \left( \frac{a}{t} \right)^4} - H(a/t,a/c) \right] \]  

\[ H(a/t,a/c) = \left\{ \begin{array}{ll}
0.89 - 0.54 & \frac{a}{c} \\
0.2 + \frac{a}{c} & 1.12 - 0.08 \frac{a}{c}
\end{array} \right\} + \left\{ \begin{array}{ll}
0.5 - \frac{1}{0.65 + a/c} + 14 \left( 1 - \frac{a}{c} \right)^2 \left( \frac{a}{t} \right)^4 & 1.12 - 0.08 \frac{a}{c}
\end{array} \right\} \]  

The superscript \( \theta \) indicates the SIF solution for the flaw subjected to uniform stressing. The SIF solutions for \( a/t = 0 \) correspond to solutions for finite crack depths, \( a, as t \to \infty \).

A similar but more complex procedure is used to evaluate the SIF for embedded flaws that are offset from a free surface of the pipe.

### 3.2 Validation of the FlawPRO Weight Function Approach

#### 3.2.1 Surface Flaws

The main validation is performed using stress variations typical of those generated at weld geometrical discontinuities by comparing the FlawPRO SIF solutions with SIFs calculated using the SC17 crack model in the computer code NASGRO (NASGRO, 2005). (NASGRO is a software program developed jointly by Southwest Research Institute® and NASA Johnson Space

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20
Center for analyzing metal fatigue and fracture. It received a 2003 R&D 100 Award. NASGRO was developed by SwRI and NASA under a 2000 Space Act Agreement and is based on a code developed by NASA beginning in the 1980s. to provide fracture control analysis for manned space programs. Among other things, NASGRO calculates the crack growth rate and remaining structural life of components undergoing cyclic loading.) The SC17 crack model is a surface flaw in a plate subjected to an arbitrary uniaxial stress variation. (The SC17 WF used in NASGRO is derived using an approach similar to that developed by Glinka and co-workers, 1991. The reference SIF solutions used in the method are based on SIF solutions generated using the boundary element method incorporated in FADD3D, 1998. FADD3D SIF solutions were also used to validate the SC17 WF.)

In the main validation calculations, the results are presented in the form of calculated SIF magnification factors, $M_k$, similar to those employed in BS 7910 (BS 7910, 1999). These factors are defined by the equation:

$$M_k = \frac{K(\text{evaluated with weld SCF})}{K(\text{evaluated with no weld})} \quad (3.15)$$

However, additional validation is first presented for the 2-D and 3-D (circular flaw) weight function used in the FlawPRO scheme by comparing the FlawPRO SIF solutions for flaws subjected to pure bending and a quadratic stress variation (in the case of the 2-D flaw) and uniform tension and pure bending (in the case of the 3-D circular flaw) with solutions available in the open literature.

The results of the validation study for 2-D flaws are shown in Figures 3.1 and 3.2, where Reference [11] is to T. K. Hellen et al. (1982) and Reference [12] is to J. P. Benthem and W. J. Koiter (1973). It can be seen from the figures that the FlawPRO 2-D SIF solutions are in excellent agreement with independently derived solutions.

![Figure 3.1: Comparison of FlawPRO SIF solutions for pure bending derived using a 2-D WF with the solution of Benthem and Koiter (1973).](image)
Figure 3.2: Comparison of FlawPRO SIF solutions for a quadratic stress variation derived using a 2-D WF with the solution of Hellen et al. (1982).

The results of the validation for the 3-D semi-circular flaws subjected to uniform and bending stressing calculated using the FlawPRO weight function scheme are presented in Figure 3.3. The validation is performed against solutions determined using the SC17 crack model in NASGRO.

Figure 3.3: Comparison of the FlawPRO SIF solutions for semi-circular flaws subjected to uniform stressing and bending with the equivalent solutions calculated using the SC17 crack model in NASGRO.

Figure 3.4 shows local stress variations typical of those expected at weld geometrical discontinuities due to nominal membrane and through-wall bend stressing. These stress fields are used in the determination of $M_k$ factors for 3-D flaws using FlawPRO and the SC17 crack model in NASGRO. The results of the $M_k$ factors for nominal membrane stressing calculated using FlawPRO and SC17 are displayed in Figure 3.5 for a range of surface flaw aspect ratios (flaw height, $h$/flaw length, $L$). The equivalent results for nominal through-wall bend stressing are shown in Figure 3.5.
Membrane Solution
OD = 20 in
t = 1.0 in
Weld cap width = 1.5 in

Through Wall Bend Solution
OD = 20 in
t = 1.0 in
Weld cap width = 1.5 in

Figure 3.4: Local membrane and through-wall bend stress variations used in the validation of FlawPRO SIF solutions. These stress fields are typical of those encountered at weld geometrical discontinuities.

It can be seen from Figure 3.5 that there is very good agreement between the FlawPRO and the SC17 results for the $M_k$ factors for membrane stressing at the deepest points on the flaws. The agreement between FlawPRO and SC17 is good for the $M_k$ factors at the surface points on the flaws, although the FlawPRO results tend to be larger than the SC17 results for deep flaws.

Figure 3.5: Comparison of $M_k$ factors for nominal membrane stressing calculated by FlawPRO with factors determined using the SC17 crack model in NASGRO.
Figure 3.6 shows the comparison between the FlawPRO and the SC17 $M_k$ factors for through-wall bend stressing. For the flaw aspect ratio of 0.5, the FlawPRO results show a “singularity” in the $M_k$ factor for the deepest point due to the definition of $M_k$ as the ratio of two SIF solutions, both of which reduce in value as the flaw depth increases and eventually have zero and then negative values. The “singularity” occurs when the SIF solution that appears in the denominator of Equation (3.15) becomes zero. Indications of this kind of behavior for the FlawPRO $M_k$ factors is also apparent at deep flaws with other aspect ratios, as shown in Figure 3.6. It should be noted that although the $M_k$ factors appear to have very high values for flaw depths near these “singularity” regions, the actual value of the SIF will be very small, as shown in Figure 2.7. Also, the singularity problem occurs at deep cracks where the remaining lives of propagating cracks are very short.

In general, it can be seen from Figure 3.6 that away from the “singularity” regions, the FlawPRO and SC17 $M_k$ factors for the deepest points on flaws are in excellent to good agreement. The agreement between the two sets of solutions is also good for the surface points on the flaws with the FlawPRO results tending to have higher values than the SC17 values for deep flaws.

Figure 3.6: Comparison of $M_k$ factors for nominal through-wall bend stressing calculated by FlawPRO with factors determined using the SC17 crack model in NASGRO.
3.2.2 Embedded Flaws

There are two embedded flaw models in FlawPRO called EC02 and EC03. These models are shown in Figure 3.7. Model EC02 is a flaw in a finite width plate whose center is offset a distance $Y$ from the nearest free surface. Model EC03 is a flaw approaching a free surface and offset from it by a distance $Y$. (Note that in the FlawPRO computer code, the offsets for these two flaw models is specified in terms of the distance, $y$, from the point on the flaw nearest the free surface to the free surface. The relationship between the two offset definitions is given by $Y = y + h/2$.) FlawPRO employs the EC02 flaw model until $h/2Y = 0.85$. Beyond this limit the EC03 model is used to calculate SIFs.

The validation of the embedded flaw models in FlawPRO is performed against the EC01 crack model in NASGRO and SIF solutions available from the open literature for EC02 and EC03. Note that the FlawPRO and NASGRO EC01 flaw models although going by a similar name are, in fact, based on different solutions. In particular, the NASGRO EC01 model is based on a closed form expression for $K$ whereas the FlawPRO model employs the weight function approach. (Note that NASGRO also contains EC02 and EC03 crack models. These are based on the same EC02 and EC03 WFs contained in FlawPRO.

In addition to validation, consistency between the EC02 and EC03 models at the limit when EC02 switches to EC03 in FlawPRO is demonstrated by comparing the SIF solutions for the two models when $h/2Y = 0.85$.

Figure 3.8 shows SIF solutions for the a-tip and c-tip positions on flaws derived using the FlawPRO and NASGRO EC01 models for centrally located flaws subjected to uniform stressing. It can be seen from the figure that there is excellent agreement between the two sets of solutions.

Figures 3.9, 3.10, and 3.11 compare the SIF solutions for the EC03 model in FlawPRO with the solutions for flaws approaching a free surface given by M. Isida and H. Noguchi (1984) for flaws offset from the surface by $h/2Y = 0.5$, $0.625$ and $0.8$, respectively, and subjected to uniform stressing. It can be seen from the figures that the SIFs at the a-tips and c-tips for the two sets of results differ by less than 5%.

Figure 3.12 shows the SIF solutions for the a-tips of embedded flaws in an infinite body subjected to uniform, linear, quadratic and cubic stress variations. The results of FlawPRO for EC02 are compared with those of Shah and Kobayashi (1971). It can be seen that there is excellent agreement between the two sets of results.

Figure 3.13 compares the EC02 and EC03 SIF solutions in FlawPRO for the crack-tip positions a', a'' and c on flaws offset from a free surface by $h/2Y = 0.85$ and subjected to uniform, linear, quadratic and cubic stress variations. The offset distance coincides with the distance in FlawPRO when the EC02 model is switched to the EC03 model when calculating SIFs. It can be seen from the figure that the EC02 and EC03 SIF solutions are consistent at the transition from one to the other to within a few percent.
Figure 3.7: Illustration of the two offset embedded flaw models in FlawPRO.

Figure 3.8: Comparison of FlawPRO (dashed lines) and NASGRO (solid lines) EC01 SIF solutions: central embedded flaw subject to uniform tension.
Figure 3.9: Comparison of FlawPRO (dashed lines) and Isida and Noguchi (solid lines) EC03 SIF solutions: embedded flaw approaching free surface (h/2Y=0.5) subjected to uniform tension.

Figure 3.10: Comparison of FlawPRO (dashed lines) and Isida and Noguchi (solid lines) EC03 SIF solutions: embedded flaw approaching free surface (h/2Y=0.625) subjected to uniform tension.
\[ \sigma(x) = \left(\frac{x}{a}\right)^i, \ i=0, 1, 2 \text{ and } 3 \]

Figure 3.11: Comparison of FlawPRO (dashed lines) and Isida and Noguchi (solid lines) EC03 SIF solutions: embedded flaw approaching free surface (h/2Y=0.8) subjected to uniform tension.

Figure 3.12: Comparison of FlawPRO EC02 SIF (dashed lines) and Shah and Kobayashi’s exact (solid lines) solutions: embedded flaw in infinite body subjected to stress variations.
\[ \sigma(x) = (x/a)^i, \quad i=0, 1 \text{ and } 2 \]

**Figure 3.13:** Comparison of FlawPRO EC02 and EC03 SIF solutions: embedded flaw subjected to stress variations approaching a free surface, \( h/2Y = 0.85 \).
4.0 NET SECTION YIELD LOADS AND REFERENCE STRESSES

4.1 General

Net section yield loads are needed in order to derive reference stresses for use in the \( J \) and \( \Delta J \) estimation schemes employed in FlawPRO. The net section yield load, \( P_o \), is the load (or combined loads in the case of simultaneously applied tension and bending) that results in yielding of the load bearing section of the pipe containing the flaw. The net section yield load is a function of the type of load applied (axial force or moment or both), yield stress, pipe dimensions, and the type and location of the flaw. The net section yield loads used in FlawPRO are derived using lower bound limit-load analysis where the externally applied loads are balanced by an internal stress field constructed so that everywhere is at yield (tension or compression) on the flawed load bearing area. Net section yield loads are required for all the flaw types available in FlawPRO: offset embedded flaws, surface flaws on the inside and outside surfaces of a pipe, and through-wall flaws.

The loads that contribute to net section yielding during reeling, installation and service are:

- pure bending (during reeling)
- combined tension and bending (during installation and service)

The pure bending during reeling arises because the spool subjects the pipe to a constant curvature equal to the radius of the spool. The combined tension and bending during installation and service can arise from membrane stressing due to axial loads, pipe bend stressing, and moments induced by membrane stressing due to axial misalignment between welded sections of the pipe. Thus welded pipes that are misaligned will be subjected to bending from axial loads even in the absence of externally applied bending. In FlawPRO, the bending due to misalignment is calculated from a misalignment pipe bend stress given by:

\[
\sigma_{\text{pipebend}}^{\text{misalignment}} = (SCF_{\text{misalignment}} - 1) \sigma_{\text{membrane}}
\]  

(4.1)

The reference stress, \( \sigma_{\text{ref}} \), is a measure of how near the pipe is to net section yielding. It is defined in terms of the net section yield load by the equation:

\[
\sigma_{\text{ref}}(a,c,a/t) = \frac{P}{P_o(a,c,a/t)} \sigma_y
\]

(4.2)

It can be seen that when the applied load equals \( P_o \) the reference stress equals the yield stress.
4.2 Reference Stresses for Combined Tension and Bend Loading

4.2.1 Embedded and Surface Flaws

The net section yield loads for surface and embedded flaws are similar in FlawPRO and are based on the flaw height and an effective flaw length, $L_{eff}$. The effective length is determined by equating the area of a rectangle with one side equal to the height of the flaw to the area of the flaw. It is given by the equation:

$$L_{eff} = \frac{\pi}{4} L$$  \hspace{1cm} (4.3)

The half angle, $\eta$, subtended by a circumferential flaw appears in the net section load expression and is given by:

$$\eta = \frac{L_{eff}}{2R_m}$$  \hspace{1cm} (4.4)

where the mean radius is given by:

$$R_m = \frac{(R_2 + R_1)}{2}$$  \hspace{1cm} (4.5)

The applied axial force $F$ is related to the membrane stress through the equation:

$$F = \pi \left(R_2^2 - R_1^2\right) \sigma_{\text{membrane}}$$  \hspace{1cm} (4.6)

and the applied moment is related to the pipe bend stress by the equation:

$$M = \frac{\pi \left(R_2^4 - R_1^4\right)}{4R_2} \sigma_{\text{pipebend}}$$  \hspace{1cm} (4.7)

Applying lower bound limit analysis (which entails equating the internal forces and moments derived from a stress distribution everywhere at yield magnitude to the externally applied forces and moments) the membrane stress at net section yielding can by derived as:

$$\sigma_{\text{net section membrane}} = \sigma_m \left[ 1 - \frac{\frac{h}{t} \eta + 2 \sin^{-1} \left( \frac{1}{2} \sin \left( \frac{h}{t} \frac{\eta}{t} \right) \right)}{\pi} \right]$$  \hspace{1cm} (4.8)
and the pipe bend stress at net section yielding is given by:

$$\sigma_{\text{pipebend}}^{\text{net section}} = \frac{4}{\pi} \sigma_y \left[ \cos \left( \frac{h \eta}{t} \right) - \frac{h}{2t} \sin(\eta) \right] \quad (4.9)$$

The membrane stress and pipe bend stresses at net section yielding under combined tension and bending satisfy the equation:

$$\frac{\pi \sigma_{\text{pipebend}}^{\text{net section}}}{4 \sigma_y} = \cos \left( \frac{1}{2} \left( \frac{\pi \sigma_{\text{membrane}}^{\text{net section}}}{\sigma_y} + \eta \frac{h}{t} \right) \right) - \frac{1}{2t} \sin(\eta) \quad (4.10)$$

Hence, the reference stress for any combination of applied membrane and pipe bend stresses is the root of the equation:

$$\frac{\pi \sigma_{\text{pipebend}}}{4 \sigma_{\text{ref}}} = \cos \left( \frac{1}{2} \left( \frac{\pi \sigma_{\text{membrane}}}{\sigma_{\text{ref}}} + \eta \frac{h}{t} \right) \right) - \frac{1}{2t} \sin(\eta) \quad (4.11)$$

At net section yielding $\sigma_{\text{ref}} = \sigma_y$ and the foregoing equation reduces to Equation (4.10).

### 4.2.2 Through-Wall Flaws

The half angle, $\eta$, subtended by a circumferential flaw is defined as follows for through-wall flaws:

$$\eta = \frac{L}{2R_m} \quad (4.12)$$

Although Equation (4.11) is applicable to embedded and surface flaws, the reference stress for a through-wall flaw under combined loading can be derived directly from it by setting $h/t=1$. It follows that the reference stress for a through-wall flaw of length $L$ is the root of the equation (for example, see Zahoor, 1989):

$$\frac{\pi \sigma_{\text{pipebend}}}{4 \sigma_{\text{ref}}} = \cos \left( \frac{1}{2} \left( \frac{\pi \sigma_{\text{membrane}}}{\sigma_{\text{ref}}} + \eta \right) \right) - \frac{1}{2} \sin(\eta) \quad (4.13)$$
5.0 J AND ΔJ ESTIMATION SCHEMES

5.1 General

The J estimation schemes employed in FlawPRO are based on a hybrid scheme that combines the Electric Power Research Institute (EPRI) approach (see V. Kumar, M. D. German, and C. F. Shih, 1981) with the reference stress approach (see R. A. Ainsworth, 1984). These two schemes are also described in Sections 9.5 and 9.6, respectively, in T. L. Anderson’s book (1995).

In FlawPRO, different J estimation schemes are used when assessing reeling, and installation and service fatigue, although both are based on the hybrid EPRI/reference stress approach. The different schemes are needed because reeling involves strain controlled loading determined by the curvature of the pipe as it is reeled on the spool, while the loading during installation and service is assumed to be load controlled.

The basis of the J formulation is that J can be written as the sum of an elastic component, $J_e$, and a fully plastic component, $J_p$:

$$ J = J_e + J_p $$

(5.1)

The elastic component is based on a first order plastically corrected flaw size that enables this term to capture the effects of crack-tip plasticity when the plastic zone size is small compared to the crack size. The fully plastic component is based on the assumption that the load bearing section containing the flaw has fully yielded such that the plastic strains totally dominate the deformation in the section. The concept behind the two component estimation scheme is illustrated in Figure 5.1. This figure shows why it is necessary to use elastic-plastic fracture mechanics (EPFM) parameters, such as J, rather than linear elastic fracture mechanics (LEFM) parameters, such as K: crack-tip plasticity increases the crack-tip driving force compared to the elastic solution and hence use of LEFM is non-conservative (not safe) for use in determining failure and/or fatigue crack growth rates in situations where significant crack-tip plasticity occurs.

In order to implement the two component J estimation scheme it is necessary to resolve the applied loads into primary (e.g. internal pressure, axial forces and moments) and secondary (e.g. residual stress) parts. This is because both primary and secondary loads contribute to the evaluation of $J_e$ but only primary loads contribute to the calculation of $J_p$. This is because, by definition, secondary loads can not influence the net section yield load because they arise from self-equilibrated stresses, and, therefore, they have no influence on fully plastic behavior. In terms of dependence on loads, the J estimation scheme can be written as:

$$ J(\sigma^p, \sigma^s) = J_e(\sigma^p, \sigma^s) + J_p(\sigma^p) $$

(5.2)

where superscripts $P$ and $S$ signify primary and secondary, respectively.
5.2 Effects of Shakedown (Elastic-Plastic Stress Relaxation) on J

In some circumstances, localized plastic relaxation and stress redistribution of stresses may occur in a FlawPRO analysis, for example, when the peak stress exceeds the yield stress. Under these circumstances, the primary stress variations used in the J formulation are always those that correspond to the elastically derived stress variations that are consistent with the forces and moments being applied to the pipe. The effects of plastic stress relaxation (hereafter called shakedown) are assumed accounted for in either a change in the secondary stress variation (e.g. relaxation of residual stresses) or, absent secondary loads, the generation of a residual stress consistent with the permanent plastic deformation induced by the local yielding.

FlawPRO carries out internal stress analyses to evaluate the effects of shakedown on elastically determined local stress variations at stress concentrators, such as weld geometrical discontinuities, where combined primary and secondary stresses exceed yield (see Section 2.0). These stress analyses are performed assuming strain controlled deformation under reeling conditions, and load controlled deformation under installation and service conditions.

In the case of strain controlled loading during reeling, the elastic-plastic stress variation consistent with the applied strain is derived. (Welding residual strains and stresses that may have been present prior to reeling are assumed wiped out by the high reeling strains.) The reeling stress is integrated to determine the value of the (strain controlled) external moment corresponding to the elastic-plastic stress variation. An elastic pipe bend stress variation consistent with the calculated external moment is determined and used as the basis of the primary stress, $\sigma_P$, appearing in Equation (5.2).

If stress concentration features are absent, the pipe bend stress induced during reeling will vary linearly through the pipe wall and across the diameter of the pipe, falling from a tensile value at the extrados where the reeling strain is a maximum to a compressive value at the intrados. If stress concentration features such as weld geometrical discontinuities are present, the variation of $\sigma_P$ through the wall of the pipe will be non-linear (see, for example, Figure 2.8) but will still integrate to balance the external moment. Hence, the form of $\sigma_P$ used in the J estimation scheme should allow for the effects of localized stress concentrations, as does the FlawPRO scheme.

Under reeling conditions, there are no self-equilibrated secondary loads at the maximum strains in the deformation cycles and so the value of $\sigma^s$ is zero. In other words, residual stresses generated during reeling are wiped out as the maximum reeling strains are applied and then regenerated again during spring-back as the pipe is straightened.

In the case of shakedown under load control, the elastic-plastic stress variation is determined for combined primary and secondary (residual) stresses, as described in Section 2.2.3. Since the elastically derived primary stress variation is assumed not to plastically relax, a shakedown modified secondary stress, $\sigma^s_{\text{shakedown}}$, is derived as the elastic-plastic stress variation for the combined loading minus the corresponding elastic stress variation. After shakedown, the load-controlled primary stress is superposed on the shakedown secondary
(residual) stress and the resulting combined stress dependence in the J estimation scheme is shown in the equation

\[ J(\sigma^p, \sigma_{shakedown}^s) = J_e(\sigma^p, \sigma_{shakedown}^s) + J_p(\sigma^p) \]  

(5.3)

5.3 J Formulation for Load Controlled Situations

5.3.1 Elastic Component of J

In the EPRI J estimation scheme, the elastic component of J is given by:

\[ J_e(a_e) = J_e(a + \phi r_y) \]  

(5.4)

In this equation, \( a_e \) is an effective flaw depth that includes a first order plastic correction, \( \phi r_y \) where

\[ \phi = \frac{1}{1 + \left( \frac{P}{P_o} \right)^2} \]  

(5.5)

\[ r_y = \frac{1}{\beta \pi} \left( \frac{K^p(a) + K^s(a)}{\sigma_y} \right)^2 \]  

(5.6)

\( \beta = 2 \) for plane stress (surface flaw tips), \( \beta = 6 \) for plane strain (embedded flaw tips), and \( K^p \) and \( K^s \) are the SIF factors evaluated for the primary and secondary stresses, respectively. In unreeled pipes the source of secondary stresses are welding residual stresses. In reeled pipes secondary stresses arise due to the residual stresses generated during the pipe straightening process as weld residual stresses are wiped out during reeling.

In terms of SIFs, \( J_e \) can be written:

\[ J_e = \frac{(K^p(a_e) + K^s(a_e))^2}{E'} \]  

(5.7)

5.3.2 Plastic Component of J

In the reference stress approach, the form of \( J_p \), the plastic component of J is given by the equation:

\[ J_p = J_e(a) V_p \left[ \frac{E \varepsilon_{ref}^p}{\sigma_{ref}} \right] = \left( \frac{K^p}{E} \right)^2 V_p \frac{E \varepsilon_{ref}^p}{\sigma_{ref}} \]  

(5.8)

where the reference stress is defined as:
\[ \sigma_{ref} = \frac{P}{P_o} \sigma_o \]  

(5.9)

and the reference plastic strain is the plastic strain corresponding to the reference stress on the true stress-true strain curve (see Figure 5.2). For a Ramberg-Osgood equation representation of the stress-strain curve which is of the form:

\[ \varepsilon = \varepsilon_o + \alpha \left( \frac{\sigma}{\sigma_o} \right)^\gamma \]  

(5.10)

the reference strain can be written as:

\[ \varepsilon_{ref}^p = \alpha \frac{\sigma_o}{E} \left( \frac{P}{P_o} \right)^\gamma \]  

(5.11)

In FlawPRO, the numerical constant \( V_p \) has a value of 1 for embedded, surface and through-wall flaws under load controlled situations. The role of this parameter in \( J \) estimation schemes is discussed by Chell et al. (1995).

### 5.4 Validation of the FlawPRO J Estimation Scheme for Flaws at Stress Concentration Features Under Load Control

The hybrid EPRI/reference stress \( J \) estimation scheme used in FlawPRO captures the effects of local stress concentration features such as occur at weld geometrical discontinuities. This is the case even when the stress at the features exceeds yield and a flaw is fully embedded in the plastic zone that develops at the feature. The effect of localized yielding at the feature on \( J \) is accounted for in the hybrid scheme by the use of the elastic stress variation for the primary loading (rather than the plastically relaxed stress variation) in the calculation of \( J_e \) and \( J_p \) together with the fact that \( J_e \) is evaluated using the effective first order plastically corrected flaw size, \( a_e \), rather than \( a \). This latter fact is the major reason why the hybrid scheme captures the effects of localized yielding at stress concentration features. For example, the failure assessment diagram (FAD) approach employed in BS 7910 does not appear to adequately allow for stress concentration features under some circumstances. This point is demonstrated in Figure 5.3. This figure shows the results of an elastic-plastic FEA to determine \( J \) for a small flaw at a notch with a high stress concentration factor subjected to uniform stressing under plane stress conditions. The computed \( J \) results are expressed in the form of a material dependent FAD consistent with the BS 7910 Level 3C approach. The failure curve in the FAD can be derived by plotting the parameter \( L_r = P/P_o = \sigma_{ref}/\sigma_y \) against \((J_e/J_e)_{ref}^{1/2}\) and is the curve against which assessments of the integrity of a flawed structure are made. In this format, Figure 5.3 shows failure curves generated using the FEA \( J \) results consistent with BS 7910 Level 3C assessment, the hybrid FlawPRO \( J \) estimation scheme, and the material dependent failure curve consistent with a BS 7910 Level 3B assessment. It is clear from the figure that the hybrid \( J \) estimation scheme is in good agreement with the FEA \( J \) solution, but that the BS 7910 Level 3B approach fails to capture the effects of the localized high stresses at the notch. Thus, in these kinds of load controlled cases, the
FlawPRO approach provides a viable and accurate method of estimating J while avoiding the complexities and effort required for a FEA J analysis.

5.5  J Formulation for Strain Controlled Reeling

5.5.1  General

Account should be taken of the strain controlled nature of reeling because, at the high strains usually associated with reeling, there can be large differences between load and strain controlled J values and hence crack-tip driving forces as flaws become larger. This point is illustrated in Figure 5.4 which compares calculated J values determined under load and strain controlled loading for a 2% applied strain. The load controlled calculations were based on the load applied to generate a 2% strain in the absence of a flaw. The strain controlled results were obtained using a procedure similar to the strain controlled J formulation used in FlawPRO and described below.

At high strains, the reference stress will be above the yield stress, so that under load control a small increase of the reference stress due to an increase in flaw size can produce a corresponding large increase in the plastic reference strain with an associated large increase in \( J_p \) and hence \( J \). This is not the case under strain controlled loading where the increase in reference strain is constrained by the boundary condition imposed in order to maintain a constant applied strain. These differences in load and strain controlled conditions result in the increasing differences in estimated J values for the two cases shown in Figure 5.4 as the flaw size increases.

The FlawPRO J estimation scheme is designed to capture the constant strain nature of reeling to avoid the large over-conservative J estimates that could arise from assuming a load controlled J formulation. This is accomplished by assuming that the reference strain remains constant, independent of flaw size, and equal to the reeling strain. It follows that the reference stress, defined as the stress on the stress-strain curve corresponding to the reference strain (see Figure 5.5), also remains constant and independent of flaw size. The hybrid J formulation used in FlawPRO to determine the crack-tip driving forces for flaws under these conditions is described in the following two sub-sections.

5.5.2  Elastic Component of J Under Strain Controlled Loading

Under strain controlled loading, the elastic component of J is given by an expression similar to the load control case, namely:

\[
J_e(a_e) = J_e(a + \phi_{reel} r_y)
\]

In this equation, \( \phi_{reel} \) is defined as:

\[
\phi_{reel} = \frac{1}{1 + \left( \frac{\sigma_{ref}}{\sigma_y} \right)^2} = \frac{1}{1 + \left( \frac{\sigma_{reel}^{el-pl}}{\sigma_y} \right)^2}
\]

\( \text{(5.13)} \)
where the reference stress is equal to $\sigma_{\text{reel}}^{\text{pl}}$, the elastic-plastic determined stress at the extrados of a reeled pipe corresponding to the reeling strain (see Figure 5.5). In addition, under reeling conditions,

$$r_y = \frac{1}{\beta \pi} \left( \frac{K^p(a, \sigma_{\text{fiber}}(x))}{\sigma_y} \right)^2$$

(5.14)

where $\sigma_{\text{fiber}}(x)$ is the elastically calculated through-wall variation in the outer fiber bend stress corresponding to the moment applied during reeling of a flaw-free pipe (see Section 2.0 for how the moment is calculated in FlawPRO). If there are no stress concentration features $\sigma_{\text{fiber}}(x)$ has the form $\sigma_{\text{fiber}}(x) = \sigma_{\text{fiber}} \left(1 - \frac{2x}{OD}\right)$, the usual pipe bend stress variation. However, if the flaw is assumed to be at a weld geometrical discontinuity then $\sigma_{\text{fiber}}(x)$ will have a non-linear variation, such as that shown in Figure 2.8.

In terms of the SIF, $J_e$ can be written:

$$J_e(a_{\text{e}}) = \frac{(K^p(a + \phi_{\text{reel}} r_y, \sigma_{\text{fiber}}(x)))^2}{E}$$

(5.15)

### 5.5.3 Plastic Component of J Under Strain Controlled Loading

Under strain controlled loading, the form of $J_p$, is given by the equation:

$$J_p = J_e(a, \sigma_{\text{fiber}}(x)) V_p \left( \frac{E \varepsilon_{\text{reel}}^p}{\sigma_{\text{reel}}^{\text{pl}}} \right)$$

(5.16)

where the numerical constant $V_p$ is set equal to 1 for embedded and through-wall flaws, and set equal to 0.9 for surface flaws. Note that the terms in the square parentheses on the right hand side of the equation remain constant at values determined for the flaw-free pipe and do not depend on flaw size.

### 5.6 Validation of FlawPRO J Estimation Scheme for Flaws in Pipes Subjected to Reeling

Three-dimensional elastic-plastic J-based FEA were performed in order to provide validation for the FlawPRO J estimation scheme under strain controlled loading. Figure 5.6 shows a typical FE model of a pipe containing a crack-like flaw subjected to simulated reeling that was used in the validation. The OD of the pipe is 219 mm and the wall thickness is 20.6 mm.
Two sets of J-based FEA calculations were performed. In the first, the pipe was “reeled” onto a spool of diameter 20 m to induce a reeling strain of 1.3% and in the second the pipe was reeled onto a spool of diameter 10 m to induce a 2.6% strain. (Note that these strain values are the nominal strains predicted by the FEA, according to mechanics of materials analysis, the strains are given by the equation $\frac{OD_{pipe}}{OD_{pipe} + OD_{spool}}$ which leads to calculated values of 1.08% and 2.14% for the 20 m and 10 m spools, respectively.)

FE modeling was used to calculate J values for circumferential surface flaws whose surface length was fixed at 20 mm but whose height (depth) varied between 1 mm and 5 mm. Calculations were performed for two stress-strain curves (see Figure 5.7), one represented by a Ramberg-Osgood equation of the form:

$$\varepsilon = \frac{\sigma}{E} + \alpha \frac{\sigma_y}{E} \left( \frac{\sigma}{\sigma_y} \right)^\gamma$$

where $\alpha=1.73$ and $\gamma=15$, and the other represented by a power law equation of the form:

$$\varepsilon = \frac{\sigma}{E}, \quad \sigma \leq \sigma_y$$

$$\varepsilon = A(\sigma - \sigma_y)^b, \quad \sigma > \sigma_y$$

where $A=0.0013$ and, $b=1.255$. In both cases, $\sigma_y=533$ MPa and $E=240,980$ MPa.

The FEA and FlawPRO J results calculated at the deepest points on flaws subjected to 1.3% straining are shown in Figure 5.8 for the two sets of stress-strain curves. Equivalent results for surface points on the flaws are not given because the computed FEA J values fell rapidly near the surface points on the flaws and it was not clear which of the values should be compared with the FlawPRO J estimation scheme values. This is a common problem in comparing crack-tip driving forces for points on flaws near free surfaces and computed values at these locations are not always considered reliable. Figure 5.9 illustrates typical J values predicted by FEA at locations on a surface flaw ranging from the deepest point to the surface point. Also shown in the figure are the results of applying the FlawPRO J estimation scheme to predict J values at the deepest and surface points. The results shown correspond to a 4 mm deep flaw in a pipe with the power law stress-strain curve subjected to 1.3% reeling strain. Although the FEA and FlawPRO J results are similar at the deepest point on the flaw, the FlawPRO surface point J estimation value is more consistent with a point on the flaw 30 degrees from the free surface. These kinds of results are typical.

Figure 5.10 shows similar FlawPRO J estimated results to Figure 5.8 for the case of 2.6% straining. However, in this case, FEA computed results were only available for a power law material and for flaw depths of 4 and 5 mm.
It can be seen from Figures 5.8 and 5.10 that the strain controlled J estimation scheme used in FlawPRO for deepest points on flaws gives results that are in good agreement with FEA results. Certainly the FlawPRO results are in better agreement with the FEA trends than would be the case if a load controlled J estimation scheme had been employed (compare the strain and load controlled J results shown in Figure 5.4). It is interesting to note that the FlawPRO scheme predicts that J values should increase almost linearly with reeling strain and this result is clearly borne out by comparison of the FEA J results for reeling strains of 1.3% and 2.6%, as shown in Figures 5.8 and 5.10, respectively.
5.7 Effects of Stress Concentration Features on J During Reeling

As for load controlled loading, the hybrid EPRI/reference stress J formulation used in FlawPRO captures the effects of local stress concentration features such as occur at geometrical discontinuities. The way this is done is illustrated in Figures 5.11 and 5.12.

Figure 5.11 shows the results of a FEA simulation of a reeled pipe subjected to a reeling strain of 3%. The pipe section is assumed to have been welded in its middle and because of axial misalignment of the centers of the two welded pipes there is a geometrical discontinuity at the weld. It has been assumed that there are no other geometrical discontinuities at the weld, such as may occur at a weld cap or weld root. Since the pipe is subjected to bending during reeling, the only effect of the misalignment is the presence of the geometrical discontinuity. (Note that unlike the case of membrane stressing where misalignment results in a pipe bend stress in a stress concentration free weld, no equivalent stresses are induced in a similar pipe subjected to pipe bend stressing.) The strain contour results shown in the inset to Figure 5.11 are color coded to show that strain is concentrated at the discontinuity: the yellow areas have strains of around the nominal reeling strain of 3%, the red areas at the discontinuity have strains of around 5.5%.

Figure 5.12 shows more details concerning the strain concentration and spatial variation in the region of the misalignment discontinuity. The same color coding is used as in Figure 5.11. In order to capture the effects of the strain concentration at the discontinuity, FlawPRO enables the user to specify an elastically determined local normalized stress variation derived for a unit applied pipe bend stress. The actual stress variation through the pipe wall is the elastically determined pipe bend stress corresponding to the moment exerted on the pipe during reeling on the spool multiplied by the normalized stress variation. This local stress variation is used in the strain controlled J estimation scheme for reeled pipes employed in FlawPRO, as shown in Figure 5.12.

Although the above example pertained to a misalignment geometrical discontinuity due to the offset of the axes of two weld pipe sections, the same principles are employed in FlawPRO to allow for the effects of weld geometrical discontinuities that can occur at weld caps and weld roots.

5.8 ΔJ Formulation for Strain Controlled Reeling

5.8.1 General

Low cycle fatigue (LCF) crack growth can occur during reeling because a single reel will generate a single closed fatigue loop with a cyclic strain slightly larger than the reeling strain. The cyclic strain is not equal to the reeling strain because of the need to compensate for elastic spring-back during straightening (see Section 2.0). Due to the large plastic strains involved in reeling, the cyclic crack-tip driving force should not be evaluated based on LEFM (i.e. ΔK). Instead, the EPFM parameter ΔJ should be used in order to avoid under-predicting the amount of LCF flaw extension. The FlawPRO expressions for ΔJ can be derived from those for J by, essentially, replacing the applied stresses by their cyclic values, and replacing the monotonic
stress-strain curve by the corresponding cyclic curve. Thus, the cyclic change in $J$ can still be expressed as the sum of elastic and plastic components through an equation of the form:

$$\Delta J = \Delta J_e + \Delta J_p$$

(5.19)

However, fatigue under reeling conditions occurs with a stress ratio, $R$, of approximately -1 corresponding to fully reversed tension and compression strain cycling. Under these conditions, crack closure effects are important as the two faces of a flaw are pressed together during the compressive part of the cycle. Closure reduces the cyclic crack-tip driving force as the force is zero while the flaw faces are in contact. To allow for this, a closure term, $U$, is introduced which has a value of around 0.5. The resulting effective cyclic crack-tip driving force $\Delta J_{eff}$ is given by the equation (for example, see McClung et al. 1994):

$$\Delta J_{eff} = \beta_s^2[U^2 \Delta J_e + U \Delta J_p]$$

(5.20)

Note that another term, $\beta_s$ has been introduced into this equation to capture the different flaw growth rates under similar cycling conditions that have been observed at the surface and deepest points of surface flaws. (It has been found that the surface points propagate at slightly lower rates than the deepest points when $R=-1$.) Thus in FlawPRO, $\beta_s$ has a value of 1 for embedded and through-wall flaws and the deepest points on surface flaws, and a value of 0.9 for the surface points on surface flaws. These values are consistent with a similar surface correction term used in the NASGRO computer program.

When determining LCF crack growth under reeling conditions, the usual Paris equation

$$\frac{da}{dN} = C\Delta K^m$$

is re-written in the form:

$$\frac{da}{dN} = C\Delta J_{eff}^{m/2}$$

(5.21)

The FlawPRO expression for $\Delta J_p$ involves terms derived from the cyclic stress-strain curve. In FlawPRO, this curve is derived from the monotonic stress-strain curve $f(\sigma)$ according to the simple rule:

$$\Delta \epsilon = 2f(\sigma), \Delta \sigma = 2\sigma.$$

(5.22)

Detailed expressions $\Delta J_e$ and $\Delta J_p$ are presented in the two sub-sections below.

5.8.2 Elastic Component of $\Delta J$ Under Strain Controlled Loading

Under strain controlled loading, the elastic component of $\Delta J$ is given by:

$$\Delta J_e(a) = \Delta J_e(a + \Delta \phi_{rec} \Delta \gamma)$$

(5.23)
where:

\[
\Delta \sigma_{\text{ref}} = \frac{1}{1 + \left( \frac{\Delta \sigma_{\text{ref}}}{\Delta \sigma_y} \right)^2} = \frac{1}{1 + \left( \frac{\Delta \sigma_{\text{ref}}^{\text{el-pl}}}{\Delta \sigma_y} \right)^2}
\]  \hspace{1cm} (5.24)

In this equation, the cyclic change in the reference stress is equal to \( \Delta \sigma_{\text{ref}}^{\text{el-pl}} \), the cyclic change in the elastic-plastic determined stress at the extrados of a reeled pipe corresponding to the cyclic change in the reeling strain. In addition,

\[
\Delta r_y = \frac{1}{\beta \pi} \left( \frac{\Delta K^p(a, \Delta \sigma_{\text{fiber}}(x))}{\Delta \sigma_y} \right)^2
\]  \hspace{1cm} (5.25)

where \( \Delta \sigma_{\text{fiber}}(x) \) is the cyclic change in the elastically calculated through-wall variation in the outer fiber bend stress corresponding to the change in the moment applied to straighten a reeled flaw-free pipe (see Section 2.0 for how the moment is calculated in FlawPRO). If there are no stress concentration features \( \Delta \sigma_{\text{fiber}}(x) \) has the form \( \Delta \sigma_{\text{fiber}}(x) = \Delta \sigma_{\text{fiber}} \left( 1 - \frac{2x}{OD} \right) \), the usual cyclic change in the pipe bend stress variation. However, if the flaw is assumed to be at a weld geometrical discontinuity then \( \Delta \sigma_{\text{fiber}}(x) \) will have a non-linear variation.

In terms of the cyclic change in SIF, \( J_e \) can be written:

\[
\Delta J_e(a_e) = \frac{(\Delta K^p(a + \Delta \phi_{\text{reel}} \Delta r_y, \Delta \sigma_{\text{fiber}}(x))^2}{E}
\]  \hspace{1cm} (5.26)

5.8.3 Plastic Component of \( \Delta J \) Under Strain Controlled Reeling

Under strain controlled cyclic loading, \( \Delta J_p \), is given by the equation:

\[
\Delta J_p = \Delta J_e(a, \Delta \sigma_{\text{fiber}}(x)) \frac{E \Delta \varepsilon_{\text{reel}}^{p}}{\Delta \sigma_{\text{reel}}^{\text{el-pl}}}
\]  \hspace{1cm} (5.27)

where the numerical constant \( V_p \) is set equal to 1 for embedded and through-wall flaws, and set equal to 0.9 for surface flaws.
Figure 5.1: Schematic showing the two component $J$ estimation scheme.

Figure 5.2: Derivation of reference strain from reference stress.
Figure 5.3: Example of flaw embedded in the plastic zone at a notch showing how the hybrid EPRI/reference stress J estimation scheme captures the effects of stress concentration features on the EPFM crack-tip driving force.

Figure 5.4: Illustration showing the effects on J of assuming flaws are subjected to load controlled or strain controlled loading during reeling.
Figure 5.5: Schematic of how the elastic-plastic stress at the extrados of a pipe during reeling is derived from the reeling strain.

Figure 5.6: Typical finite element model used to compute $J$ values for circumferential surface flaws in pipes subjected to reeling.
Figure 5.7: Comparison of the two stress-strain curves used in validation of the FlawPRO J estimation scheme for strain controlled reeling.

Figure 5.8: Comparison of FEA J results for the deepest points on surface flaws with the predictions of the FlawPRO J estimation scheme for pipes subjected to 1.3% reeling strain. The comparison is made for two stress-strain behaviors, Ramberg-Osgood and power law, see Figure 5.7.
Figure 5.9: Illustration of how the FlawPRO J estimation scheme tended to predict J values for surface points on flaws that were more consistent with the FEA J values for points on the flaws at around 30 degrees from the surface.

Figure 5.10: Comparison of FEA J results for the deepest points on surface flaws with the predictions of the FlawPRO J estimation scheme for pipes subjected to 2.6% reeling strain. The comparison is made for power law stress-strain behavior.
Figure 5.11: FEA results showing the strain concentration of a reeled pipe subjected to a 3% reeling strain containing a geometric discontinuity due to the welding together of axially misaligned pipes. The strains in the yellow area in the inset correspond approximately to the nominal reeling strain of 3% while the strains in the red area at the discontinuity have values of around 5.5%.
Figure 5.12: In order to allow for the strain concentration at the misalignment geometric discontinuity (illustrated on the left), FlawPRO uses a normalized stress variation (example shown on right) that captures the effects of the discontinuity on the elastically determined pipe bend stress. The resulting elastically calculated through-wall variation in the outer fiber bend stress \( \sigma_{\text{fiber}}(x) \) corresponding to the moment applied during reeling, is used in FlawPRO to evaluate \( J \), as shown in the equation in the figure.

\[
J = J_c(a_e, \sigma_{\text{fiber}}(x)) + J_p(a, \sigma_{\text{fiber}}(x))
\]
6.0 MECHANICS OF CRACK GROWTH

6.1 Cyclic Crack Growth

6.1.1 General

The FlawPRO approach to evaluating fatigue crack extension is based on the Paris equation. In its simplest form, this equation is written as:

\[ \frac{dc}{dN} = C\Delta K^m \]  

(6.1)

This equation is numerically integrated for each of the load steps that form a load spectrum to obtain the increment of crack extension that occurs during a single application of the spectrum. This procedure is straight forward for flaws that have a single degree of freedom, such as through-wall flaws, when the length \( L=2c \).

In the case of surface flaws that have two degrees of freedom, the extensions that occurs at the deepest and surface points are not independent but are linked by the dependence of \( \Delta K \) on the shape and size of the flaw. Evaluation of crack growth for these kinds of flaws involves the integration of the coupled equations:

\[ \frac{da}{dN} = C\Delta K^m_a(a,c), \frac{dc}{dN} = C\Delta K^m_c(a,c) \]  

(6.2)

where subscripts \( a \) and \( c \) signify the a-tip (deepest point) and c-tip (surface points), respectively, and the flaw height \( h=a \) and the flaw length \( L=2c \). After an increment of flaw growth the change in the flaw size is given by:

\[ a_{new} = a + da, \quad c_{new} = c + dc, \quad h_{new} = a_{new}, \quad L_{new} = 2c_{new} \]  

(6.3)

where the subscript \( new \) signifies the new flaw size.

In the case of offset embedded flaws that have three degrees of freedom, the extensions that occurs at the two points on the flaw nearest the two free surfaces (the \( a^- \)-tip and \( a^+ \)-tip, respectively), and the points midway between these points (the c-tips) are evaluated by integrating three coupled equations of the form:

\[ \frac{da^-}{dN} = C\Delta K^m_a(h,c,y), \frac{da^+}{dN} = C\Delta K^m_a(h,c,y), \frac{dc}{dN} = C\Delta K^m_c(h,c,y) \]  

(6.4)

where superscripts \(-\) and \(+\) signify the a-tip nearest a free surface and the a-tip furthest from the same free surface, respectively, and \( y \) is the offset distance. In these equations, the flaw height \( h=(a^- +a^+)/2 \), and after an increment of flaw growth the change in the flaw size is given by:
Two different fatigue crack growth regimes occur in pipes that enter service after reeling. The first regime occurs under low cycle fatigue (LCF) conditions. For a single reel installation, the LCF is associated with two bending operations (reeling the pipe around a spool and then around the aligner) and two straightening operations (pulling the pipe off the spool and then off the aligner). It is assumed in FlawPRO that the strains induced by reeling the pipe around the spool and around the aligner are the same. Based on this assumption, a pipe will experience one completed low cycle strain loop on the first reel, and two completed loops on subsequent reels. Due to the high strains produced during reeling, the extrados of the pipe (which experiences the peak straining) will undergo tensile and compressive yielding which results in LCF at a stress ratio of approximately $R=-1$. More details concerning the stress analysis aspects of reeling are provided in Section 2.0.

The high cyclic strains generated during reeling will subject pre-existing flaws to large scale reversed plastic deformation. Under these conditions, it is non-conservative to use LEFM based crack-tip driving forces (e.g. $\Delta K$) to evaluate fatigue crack extension and the EPFM parameter $\Delta J$ has to be used instead. The reversed load cycling will force the faces of the flaws to come into contact during the compressive half of the LCF cycle, resulting in crack closure and a reduction in the cyclic crack-tip driving force.

In the second fatigue regime, which occurs during installation (when the pipe may be suspended overboard and be subjected to wave motion) and during service, a pipe will experience cyclic strain changes that remain elastic while the mean stress acting on the pipe is high. Under these conditions, pre-existing flaws will be subjected to small scale cyclic yielding at high stress ratios where the flaw faces remain apart and crack closure does not occur. In this loading regime it is appropriate to use cyclic crack-tip driving forces based on the LEFM parameter $\Delta K$.

The way FlawPRO determines crack extension in these two fatigue regimes is described in more detail in the two sub-sections below.

### 6.1.2 Fatigue Crack Growth (Reeling)

In order to account for cyclic plasticity during reeling, the Paris equation given above is re-expressed in terms of the EPFM $\Delta J$ and, for a one degree of freedom flaw, takes the form:

$$\frac{dc}{dN} = C' \Delta J^{m/2}$$  \hspace{1cm} (6.6) \hspace{1cm} 

In the small scale cyclic yielding regime where LEFM is applicable:
\[ \Delta J = \frac{\Delta K^2}{E'} \]  

so the foregoing equation reduces to the equivalent Paris equation in this operating regime with 
\[ C' = (E')^{m/S} C. \]  
In general, a plastically corrected value of \( \Delta K \) can be defined in terms of \( \Delta J \) by the equation:

\[ \Delta K^{el-pl} = \sqrt{E'} \Delta J \]  

Because of crack closure effects during reeling and surface interactions that reduce the flaw propagation rate at surface points with respect to the deepest points on a flaw, the equation used in FlawPRO to determine crack growth rates is modified to read:

\[ \frac{dc}{dN} = C' \Delta J^{m/2} \]  

where the subscript \( eff \) signifies effective. The use of an effective cyclic crack-tip driving force is illustrated in Figure 6.1. It can be seen from the results presented in this figure that \( \Delta K_{eff} \) collapses fatigue crack growth rate curves measured at different stress ratios (and hence with different crack closure effects) onto a single curve. More details concerning the definition of \( \Delta J_{eff} \) are given in Section 5.8.

Validation for the approach of replacing \( \Delta K \) by \( \Delta J^{1/2} \) in the Paris equation in order to capture the effects of cyclic plasticity on crack growth rates is provided in Figure 6.2. This figure presents data that includes measurements of fatigue crack growth rates in the LEFM and EPFM regimes. It can be seen that when the growth rates are plotted against \( \Delta J \) nearly all the data points fall within the same parallel straight lines (in log-log space) that pass through regions where flaw extensions are under LEFM and EPFM conditions.

Typically, during reeling, calculated crack closure corrected values of \( \Delta K^{el-pl} \) exceed 100 ksi in\(^{1/2} \) (110 MPa m\(^{1/2} \)) that correspond to \( \Delta J \) values shown in Figure 6.2 of around 300 in-lb/in\(^2 \) (0.058 MPa-m) and growth rates of around 0.01 in/cycle (0.25 mm/cycle). Although these values are ball park numbers, they illustrate that fatigue crack extension from reeling could be of the same order as flaw extension from ductile tearing.
Figure 6.1: Crack growth rates in steels appear to be R-dependent when plotted against $\Delta K_{\text{total}}$ (= $\Delta K$) (left figure) but collapse onto a single curve when plotted against a closure corrected parameter, $\Delta K_{\text{eff}}$ (right).

Figure 6.2: Example of results showing $\Delta J$ correlates fatigue crack growth rates under cyclic linear elastic and cyclic elastic-plastic crack-tip conditions (see N.E. Dowling, 1976).
6.1.3 Fatigue Crack Growth (Installation and Service)

Fatigue during installation and service occurs at relative low $\Delta K$ values (circa 25 ksi in $^{1/2}$ and below) compared to the very high $\Delta K_{el-pl}$ values (> 100 ksi in $^{1/2}$) that can occur during reeling. Hence, the description of flaw growth in FlawPRO under installation and service fatigue covers flaw propagation rates that range from the very low (including zero) to intermediate values. This is accomplished by dividing the crack growth rate curve into three regions determined in terms of $\Delta K$ where the rates in each region are mathematically described using Paris-type equations. The three regions are labeled threshold, where zero growth occurs because $\Delta K < \Delta K_{th}$, low, which describes growth rates just above threshold where $\Delta K_{th} < \Delta K < \Delta K_{trans}$, and high, which describes intermediate rates where $\Delta K > \Delta K_{trans}$. This approach is illustrated in Figure 6.3. The subscript trans signifies the value at the transition from low growth (Region 2) to high growth (Region 3). Mathematically, the approach is equivalent to:

\[
\begin{align*}
\frac{dc}{dN} &= 0, \quad \Delta K < \Delta K_{th} \quad \text{(Region 1: below the cyclic threshold)} \\
\frac{dc}{dN} &= C\Delta K^n, \quad \Delta K_{th} \leq \Delta K < \Delta K_{trans} \quad \text{(Region 2: between threshold and transition)} \\
\frac{dc}{dN} &= B\Delta K^n, \quad \Delta K_{trans} \leq \Delta K \quad \text{(Region 3: above transition)}
\end{align*}
\]

(6.9)

Figure 6.3: The $dc/dN$ curve is represented in FlawPRO by three regions: Region 1 (below threshold, no growth), and low (Region 2) and high (Region 3) $\Delta K$ growth regions, as shown in the figure.
6.2 Ductile Tearing During Reeling

The amount of ductile tearing, $\Delta a_i$, that occurs during reeling for flaw depth $a$ is solved as the root of the equation:

$$J(a + \Delta a_i) = J_R(\Delta a_i)$$

(6.10)

The solution method is shown graphically in Figure 6.4.

![Figure 6.4: Schematic showing how the amount of crack-tip blunting and ductile tearing is determined in FlawPRO during reeling.](image)

It is assumed in FlawPRO that the amount of ductile tearing during reeling is small compared to the flaw size. This enables the tear length at each crack-tip position to be calculated based on the flaw dimensions before tearing. The final flaw size after tearing is determined using the increments of tearing calculated for each degree of freedom and equations similar to those employed to evaluate fatigue crack growth (compare Equations (6.3), (6.4) and (6.5)).

It can be seen that a solution to Equation (6.10) is only possible provided the applied J curve intersects the $J_R$ curve. If the applied J falls above the $J_R$ curve then the flaw is predicted to be unstable since the applied crack-tip driving force (J) will always exceed the material’s resistance to crack propagation ($J_R$). The point of incipient instability is predicted when the applied J curve is tangential to the $J_R$ curve. For these instability conditions to occur, the following equations must be simultaneously satisfied:
\[ J(P_{\text{inst}}, a_{\text{inst}} + \Delta a_i) = J_R(\Delta a_i) \]  

(6.11)

\[ \frac{dJ(P_{\text{inst}}, a + \Delta a_i)}{da} \bigg|_{a=a_{\text{inst}}} = \frac{dJ_R(\Delta a_i)}{d(\Delta a_i)} \]

where subscript \textit{inst} signifies quantities evaluated at incipient instability.

6.3 Tear-Fatigue During Reeling

Tear-fatigue is a phenomenon that occurs when flaws are subjected to cyclic load changes while the applied \( J \) at maximum load exceeds, \( J_{lc} \), the \( J_R \) value at the initiation of ductile tearing. The result is a synergy in the crack growth mechanisms associated with tearing and fatigue that produces an enhancement in measured flaw propagation rates compared to those predicted using the Paris equation. Figure (6.5) illustrates that the enhancement in cyclic growth rates due to tear-fatigue interactions can raise fatigue rates by one or two orders of magnitude compared to the rates predicted by applying a Paris type of equation.

![Graph showing tear-fatigue](image)

**Figure 6.5:** Results of fatigue crack growth rate measurements on small specimens subjected to load control showing the enhancement in growth rates at high \( \Delta K_{\text{eff}} \) values due to tear-fatigue. Although no cyclic plasticity occurred in the tests the specimens were loaded beyond net section yielding at the maximum load.

The synergy between ductile tearing and LCF during reeling is captured in FlawPRO by using a procedure that is equivalent to replacing the Paris equation by the following (G. G. Chell, 1984):
\[
\frac{da}{dN}_{\text{tear-fatigue}} = \frac{\frac{da}{dN}_{\text{Paris}}}{1 - \frac{dJ_{\text{max}}}{dJ_{R}} - \frac{da}{d(\Delta a)}}
\] (6.12)

where the subscripts are self-explanatory. It can be seen that the foregoing equation predicts that the tear-fatigue crack growth rate will become infinite (i.e. flaw instability will occur) when the denominator equals zero, which is consistent with the condition for ductile instability predicted given by Equation (6.11) with \(a\) interpreted as the flaw depth equal to the initial flaw depth plus the sum of all the increments of fatigue crack growth that have occurred. This is called the Memory Model for tear-fatigue since it implies that a material’s resistance to ductile fracture (characterized by its \(J_{R}\) curve) has a memory of previous tearing events during tear-fatigue crack growth. The model is different from assuming No Memory Model behavior, where after each cyclic load change is completed the increment of tearing that occurs is evaluated using Equation (6.10) and is assumed independent of any previous tearing events. The difference between the Memory Model and the No Memory Model is highlighted by the fact that in the Memory Model the combined flaw extension due to tear-fatigue, \(\frac{da}{dN}_{\text{tear-fatigue}}\), is, as shown by Equation (6.12), independent of the value of \(J\) at maximum load in the cycle, \(J_{\text{max}}\) and \(J_{R}\) but, instead, is a function only of the gradients of these two quantities.

The steps involved in flaw growth by tear-fatigue based on the Memory Model are illustrated in Figure 6.6.

**Tear-Fatigue Process**

1. Before fatigue: crack is stable and \(J = J_{R}\)
2. Crack grows by fatigue and \(J_{R}\) Curve curve moves with crack tip
3. Crack tears until it is stable and \(J = J_{R}\)
4. Crack advances by another fatigue cycle and process is repeated

*Figure 6.6: Illustration of flaw growth by tear-fatigue based on the Memory Model.*
6.4 Validation of the Memory Model Based Tear-Fatigue Model in FlawPRO

The predictions of Equation (6.12) have been validated through measurements of crack propagation rates in laboratory specimens loaded under conditions that induced tear-fatigue crack growth. The results of these tests are displayed in Figure 6.7 where the left hand figure shows tear-fatigue propagation rates predicted using a Paris equation plotted against measured rates, and the right hand figure shows the predictions of the tear-fatigue model incorporated into FlawPRO plotted against the measured tear-fatigue crack growth rates. It is clear from these figures, that the Paris equation under-predicts the measured growth rates by an order of magnitude or more, whereas the tear-fatigue model used in FlawPRO accurately predicts the acceleration in rates due to the synergy between ductile tearing and fatigue crack growth.

Equation (6-12) provides another means of validating the Memory Model of tear-fatigue incorporated into FlawPRO. This equation predicts that Paris equation crack growth rates that do not include a tear-fatigue enhancement will occur when tear-fatigue tests are carried out with \( \frac{dJ_{\text{max}}}{da}=0 \), irrespective of the value of \( J_{\text{max}} \). Thus, the model predicts Paris equation growth rates will be recovered if a load mode change is made during a tear-fatigue test so that \( \frac{dJ_{\text{max}}}{da}=0 \) when previously tear-fatigue had been observed because \( \frac{dJ_{\text{max}}}{da}>0 \). The results shown in Figure 6.8 demonstrate that this is the case. The top figure shows a displacement controlled mode of cyclic loading that results in an increasing value of \( J_{\text{max}} \) and tear-fatigue because \( \frac{dJ_{\text{max}}}{da}>0 \). However, after some cycles, the load mode is changed from displacement control to a control based on the cyclic change in the area under the load displacement curve that

Figure 6.7: The Paris equation fails to predict crack growth rates under tear-fatigue conditions (left figure). The tear-fatigue model incorporated into FlawPRO based on the Memory Model successfully predicts the observed crack growth rates (right figure). (Reference: K. J. Nix, N. Knee, T. C. Lindley and G. G. Chell, 1988.)
results in the value of $J_{\text{max}}$ remaining constant (i.e. $dJ_{\text{max}}/da=0$) in subsequent cycling. The applied $J_{\text{max}}$ at the mode change was around $10J_{Ic}$, where $J_{Ic}$ is the value at the initiation of ductile tearing. The lower figure in Figure 6.8 shows that the high fatigue crack propagation rates that are a characteristic of tear-fatigue are suddenly reduced to rates more consistent with Paris equation predictions after the load mode change is made, even though the applied $J_{\text{max}}$ is greatly above the value needed to initiate ductile tearing, validating the Memory Model used in FlawPRO.

Figure 6.8: In agreement with observed behavior, the tear-fatigue model in FlawPRO predicts no acceleration in fatigue rates relative to Paris equation predictions when the applied $J$ versus crack depth curve ($dJ/da$) has zero slope due to a load mode change during testing. (Reference: K. J. Nix, N. Knee, T. C. Lindley and G. G. Chell, 1988.)
7.0 CRACK TRANSITIONING

7.1 General

The crack transitioning capability in FlawPRO enables advantage to be taken of the extended reeling and service lives that result when, for example, an embedded crack transitions to a surface crack that can then transition into a through-wall crack. It is important to note that crack transitioning should only be used in FlawPRO when the pipe material displays ductile failure behavior as cracks in brittle materials may become unstable during transitioning.

7.2 Criteria for Initiating Crack Transitioning

Crack transitioning may occur during FlawPRO flaw growth calculations due to a number of reasons. These are listed in Table 7.1.

Table 7.1: Crack transitioning criteria used in FlawPRO.

<table>
<thead>
<tr>
<th>Possible Causes of Crack Transitioning Being Initiated in FlawPRO</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>The applied crack-tip driving force exceeds the material toughness.</td>
<td>This condition can arise during either installation or service and may result from the application of the maximum stress in a load step, or the application of the worst-case stress at the end of every period.</td>
</tr>
<tr>
<td>The fatigue crack growth rate is so high that this rate times the number of cycles in a load step exceeds the remaining un-cracked ligament in the pipe wall.</td>
<td>When the crack growth rate becomes large this indicates that there is very little life remaining.</td>
</tr>
<tr>
<td>The number of fatigue integration steps needed to evaluate the crack growth during a load step exceeds 1000.</td>
<td>The number of integration steps to evaluate the crack extension for a load step is increased as the growth rate increases in order to avoid inaccuracies in the growth calculations due to too large a crack growth increment. When the number of integration steps becomes very large, this indicates extremely fast crack propagation rates and that there is little life remaining.</td>
</tr>
<tr>
<td>The crack size exceeds the maximum size for which the fracture mechanics solutions are valid. (See Table 7.2 for the range of applicability of the FlawPRO SIF solutions.)</td>
<td>The stress intensity factor solutions contained in FlawPRO are only valid for cracks within restricted size ranges. Crack transitioning is assumed when these ranges are exceeded because it is unsafe to extrapolate these solutions beyond the crack size ranges for which they are valid.</td>
</tr>
<tr>
<td>The amount of ductile tearing exceeds the maximum or saturation tear length.</td>
<td>This condition is applicable when ductile tearing occurs during reeling and straightening. A maximum tear or saturation length is specified in FlawPRO and should be based on the maximum amount of tearing observed when measuring the J-Resistance curve. Crack transitioning is assumed in FlawPRO when the predicted tearing at a flaw exceeds the maximum allowable amount because it is unsafe to extrapolate the J-Resistance curve beyond this length.</td>
</tr>
</tbody>
</table>
Table 7.2: Ranges of applicability of FlawPRO SIF solutions.

<table>
<thead>
<tr>
<th>Flaw Type</th>
<th>(OD/t) Range</th>
<th>(h/t) or (h/y) Range</th>
<th>(h/L) Range</th>
</tr>
</thead>
</table>
| Embedded Flaw Offset \(y\) from Outside of Pipe | No Limitation (Solution Based on Embedded Flaw in Plate) | \(\max\left(\frac{h}{h + 2y}, \frac{h}{2t - 2y - h}\right) \leq 0.9\) | \(\frac{h}{L} \leq 2\)  
\(\frac{h}{L} = 2\) solution  
used when \(\frac{h}{L} > 2\) |
| Embedded Flaw Offset \(y\) from Inside of Pipe | No Limitation (Solution Based on Embedded Flaw in Plate) | \(\max\left(\frac{h}{h + 2y}, \frac{h}{2t - 2y - h}\right) \leq 0.9\) | \(\frac{h}{L} \leq 2\)  
\(\frac{h}{L} = 2\) solution  
used when \(\frac{h}{L} > 2\) |
| Surface Flaw on Outside of Pipe | \(\frac{OD}{t} \leq 200\)  
\(\left(\frac{OD}{t}\right) = 200\) solution  
used when \(\left(\frac{OD}{t}\right) > 200\) | \(\frac{h}{t} \leq 0.9\) | \(\frac{h}{L} \leq 1\)  
\(\frac{h}{L} = 1\) solution  
used when \(\frac{h}{L} > 1\) |
| Surface Flaw on Inside of Pipe | \(8 \leq \frac{OD}{t} \leq 200\)  
\(\left(\frac{OD}{t}\right) = 8\) solution  
used when \(\left(\frac{OD}{t}\right) < 8\)  
\(\left(\frac{OD}{t}\right) = 200\) solution  
used when \(\left(\frac{OD}{t}\right) > 200\) | \(\frac{h}{t} \leq 0.9\) | \(\frac{h}{L} \leq 1\)  
\(\frac{h}{L} = 1\) solution  
used when \(\frac{h}{L} > 1\) |
| Through Wall Flaw | No Limitation                        | \(\frac{h}{\pi OD} \leq 0.9\) | Not Applicable |
7.3 Types of Transitions

The crack transitions that can occur are dependent on the type of crack, as shown in Table 7.3.

Table 7.3: Crack transitioning possibilities.

<table>
<thead>
<tr>
<th>Initial Crack Type</th>
<th>Possible Crack Type after First Transition</th>
<th>Crack Type after Second Transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Embedded</td>
<td>Surface crack emanating from the outside of the pipe</td>
<td>Through-wall crack</td>
</tr>
<tr>
<td>Embedded</td>
<td>Surface crack emanating from the inside of the pipe</td>
<td>Through-wall crack</td>
</tr>
<tr>
<td>Surface Crack on Outside of Pipe</td>
<td>Through-wall crack</td>
<td>None</td>
</tr>
<tr>
<td>Surface Crack on Inside of Pipe</td>
<td>Through-wall crack</td>
<td>None</td>
</tr>
<tr>
<td>Through-wall crack</td>
<td>None</td>
<td>None</td>
</tr>
</tbody>
</table>

7.4 Sizes of Flaws after Transitioning

In FlawPRO, the size of a re-characterized flaw after transitioning is based on the procedures incorporated in the computer code DARWIN™ (Design Assessment of Reliability With INspection). This software design code, developed for the Federal Aviation Administration (FAA) by Southwest Research Institute (SwRI) to help turbine engine manufacturers improve the safety of commercial airliners, was named one of the 100 top technical achievements of the past year by R&D Magazine. DARWIN™ was developed in cooperation with commercial gas turbine engine manufacturers Rolls-Royce, Pratt & Whitney, Honeywell, and GE Aircraft Engines, and the Rotor Integrity Subcommittee (RISC) of the Aerospace Industries Association. The software code is used to assess the risk that a turbine engine’s rotor disk might contain a dangerous metallurgical or manufacturing flaw that could cause fatigue cracking, leading to possible catastrophic failure.

The crack transitioning criterion for embedded to surface flaws is illustrated in Figure 7.1. This figure shows the shape and size of an embedded flaw before and after it has transitioned to a surface flaw. The flaw dimensions after transition (height, $a_{\text{new}}$, and length, $L_{\text{new}}=2c_{\text{new}}$) are shown expressed in terms of the flaw dimensions before transition (height, $2a$, length, $L=2c$, and offset $y$). The size of the surface flaw after transitioning is based on the area of the embedded flaw and the area between the flaw and the surface it transitions to i.e. the area enclosed by the two vertical lines shown on the left of Figure 7.1.

The length, $L_{\text{new}}$, of a through-wall flaw after it has transitioned from a surface flaw of surface length $L=2c$ is given by $L_{\text{new}}=L$. 
7.5 Effects of Transitioning on Fatigue Crack Growth Rates

Besides a re-characterization of the crack type, crack transitioning can also result in a change in the fatigue crack growth rate when embedded flaws break through to either the inside or outside surfaces and become exposed to the environments adjacent to those surfaces. This possibility is allowed for in FlawPRO by enabling the user to specify the fatigue crack growth rate equations that are appropriate for embedded, outer surface, and inner surface flaws. These equations are used to determine the crack propagation rates for embedded flaws after they transition to surface flaws.
8.0 CONCLUSIONS

The program FlawPRO contains advanced EPFM methods and more conventional LEFM based approaches that enable flaw extension under the high strain conditions pertaining during reeling and crack growth under the nominally elastic strain conditions pertaining during installation and service to be predicted. The EPFM methods are based on the crack-tip driving forces, \( J \) and \( \Delta J \), and the LEFM methods are based on \( K \) and \( \Delta K \). It has been demonstrated that the \( J \) estimation schemes employed in FlawPRO are appropriate for strain controlled and load controlled situations, and for flaws emanating from stress concentration features such as those associated with weld geometrical discontinuities. It has also been demonstrated that the mechanistic model incorporated in FlawPRO for describing the synergy between ductile tearing and LCF is appropriate for predicting crack growth during reeling. Thus FlawPRO contains all the technical elements required for performing engineering critical assessments of reeled and conventionally installed sub-sea pipes.

Much of advanced methodology incorporated in FlawPRO for assessing flaw growth during reeling is not included in standard engineering critical assessment procedures such as BS 7910:1999. In particular, these procedures give no guidance on the determination of the parameters \( J \) and \( \Delta J \) under strain controlled loading and the mechanics of treating tear-fatigue, especially under low cycle fatigue conditions. Both these technical issues are crucial to an accurate and physically meaningful assessment of flaw extension during reeling.

9.0 ACKNOWLEDGMENTS

The full-scale validation of FlawPRO\(^{TM}\), as well as certain software enhancements, were performed under a Joint Industry Program (JIP) entitled “Validation of a Methodology for Assessing Defect Tolerance of Welded Reeled Risers,” which was conducted by SwRI for the offshore industry. SwRI is pleased to acknowledge the advice, encouragement, and financial support received from the following JIP member companies: ChevronTexaco, ExxonMobil, Shell, Technip, Tenaris, and Total. Special acknowledgements are also due to Paulo Gioielli and Jaime Buitrago of ExxonMobil who championed the JIP formation, as well as to Frans Kopp and Bruce Miglin of Shell who had the vision to initially support the development of FlawPRO.
10.0 REFERENCES


