

CAPABILITIES AND APPLICATIONS OF PROBABILISTIC METHODS IN FINITE ELEMENT ANALYSIS

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Abstract

The ability to quantify the uncertainty of complex engineering structures subject to inherent randomness in loading, material properties, and geometric parameters is becoming increasingly important in the design and analysis of structures. Probabilistic finite element analysis provides a means to quantify the reliability of complex systems in such areas as aerospace and automotive structures. However, for wide acceptance, probabilistic methods must be interfaced with widely used commercial finite element solvers such as ABAQUS, ANSYS, and NASTRAN. In addition, the finite element results are generally post-processed to evaluate a measure of useful life with tools that can compute quantities such as fatigue life. In this paper, interfacing and performance issues involved in coupling probabilistic methods, commercial finite element solvers and post finite element solvers are discussed. An example problem consisting of an integration of the NESSUS probabilistic analysis software, the ABAQUS finite element solver and fatigue life assessment is used to demonstrate concepts.

1. Introduction

Numerical simulation is now routinely used to predict the behavior and response of complex systems, especially when consideration of nonlinear effects, multiple physics, or complex geometry is required. The use of computational simulation is relied upon increasingly more as performance requirements for engineered structures increase and as a means of reducing testing. Since structural performance is directly affected by uncertainties

associated with models or in physical parameters and loadings, the development and application of probabilistic analysis methods suitable for use with complex numerical models is needed. An established method of accounting for uncertainties is based on a factor-of-safety approach. However, this approach generally leads to overconservative and hence uneconomical designs. Furthermore, the maximum potential of the design is never realized and the reliability is not quantified.

The traditional method of probabilistic analysis is Monte Carlo simulation. This approach generally requires hundreds of thousands of simulations to calculate the low or high probabilities, and is impractical when each simulation involves extensive finite element computations. Therefore, more efficient methods are needed. Approximate fast probability integration (FPI) methods have been shown to be many times more efficient than Monte Carlo simulation [1]. The advanced mean value (AMV) procedure, based on FPI, can predict the probabilistic response of complex structures with relatively few response calculations. An additional benefit of AMV is the computation of probabilistic sensitivity measures. More efficient sampling techniques have been developed such as adaptive importance sampling procedures [2].

Southwest Research Institute (SwRI) has been addressing the need for efficient probabilistic analysis methods and interfacing to commercial finite element analysis packages for over fifteen years, beginning with the development of the NESSUS probabilistic analysis computer program.

NESSUS can be used to simulate uncertainties in loads, geometry, material behavior, and other user-defined uncertainty inputs [3] to compute probability of failure and probabilistic sensitivity measures. NESSUS has a built-in finite element structural modeling capability as well as interfaces to many commercially available finite element programs. NESSUS was initially developed by a team led by SwRI for the National Aeronautics and Space Administration (NASA) to assess uncertainties in critical space shuttle main engine components [4].

2. Probabilistic Analysis Methods

Methods for performing probabilistic analysis include sampling methods, most probable point (MPP) methods, and hybrid methods (combined sampling and MPP) among others. MPP methods calculate the probability of failure by first locating the MPP and then applying numerical integration techniques. Locating the MPP generally requires an iterative solution based on sensitivities of the response with respect to each of the random variables. The key to efficiency in using an MPP-based method is locating the MPP as quickly as possible. MPP methods are well developed for problems with single, well-behaved (monotonic and continuously differentiable) limit states. Problems with ill-behaved limit-states or with multiple limit-states, however, generally require the use of sampling.

Sampling methods such as Monte Carlo or Latin Hypercube Simulation (LHS) repeatedly evaluate the deterministic model to generate samples of the model response, from which the response statistics and probability of failure are approximated. A major advantage of sampling methods is that the deterministic model does not need to be simplified or approximated to perform the probabilistic analysis. Thus, problems with multiple failure modes and/or ill-behaved response functions can be solved without difficulty. A significant disadvantage of sampling is that the probabilistic analysis can become quite costly due to the large number of samples required to compute small failure probabilities.

MPP-based methods approach the problem differently than sampling methods. If X represents a vector of N random variables, a limit-state function can be defined by the equation $g(X) = 0$, where g is some performance measure. Usually g is formulated

as $g(X) = Z(X) - z_0 = 0$, where $Z(X)$ is a model response and z_0 is a particular response value. The limit-state represents a hypersurface in N dimensional random variable space that separates the space into failure and non-failure regions. Failure is defined when $g(X) \leq 0$. The probability of failure, p_f , is the integration of the joint probability density function (JPDF) $f_x(X)$ over the failure region Ω :

$$p_f = \int_{\Omega} f_x(X) dX \quad (1)$$

The solution of Eqn. (1) is seldom possible in closed form and thus Monte Carlo simulation or FPI techniques are used. The MPP, limit-state, and JPDF are illustrated in Figure 1.

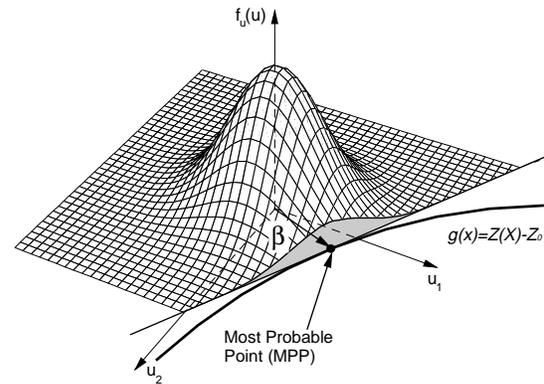


Figure 1. JPDF and MPP for Two Random Variables

When $Z(X)$ is expensive to evaluate, an approximate first-order Taylor series can be developed

$$Z(X) = a_0 + \sum_{i=1}^N a_i X_i \quad (2)$$

where a_i are the structural sensitivities computed using perturbation [5] or finite difference methods.

For each Z_0 , FPI methods locate the MPP using the approximate $Z(X)$ given by Eqn. (2) and the JPDF. The CDF established using Eqn. (2) is termed the mean value (MV) first-order method. An improvement to the MV solution can be made by substituting the MPP values of the random variables into $Z(X)$ and calculating an updated response. This procedure is termed the advanced mean value (AMV), first-order method. The AMV+ method continues from the AMV step by iteratively recreating the approximate function about the predicted MPP until convergence. AMV+ provides

a locally accurate approximate function about the MPP for probability calculations and has been shown to be accurate and efficient for many problems [6].

3. Interfacing with Commercial Finite Element Programs

A number of issues arise when performing probabilistic analysis with commercially finite element (FE) analysis packages. These issues are described in the following sections for the ABAQUS FE analysis program [7] but are equally applicable to other packages such as ANSYS or NASTRAN.

NESSUS Interface to ABAQUS

The NESSUS interface to ABAQUS integrates the probabilistic analysis capabilities of the NESSUS program with the deterministic structural analysis capabilities of the ABAQUS FE code.

The NESSUS code includes the NESSUS/PFEM module for coordinating the probabilistic algorithms with the deterministic capabilities of a FE analysis code. The NESSUS code provides the mean value (MV), advanced MV (AMV), the iterative AMV (AMV+), adaptive importance sampling (AIS), Latin Hypercube simulation, Monte Carlo sampling, and response surface (RSM) probabilistic analysis methods. An overview of these methods is given in the NESSUS User's Manual [3]. The capabilities of the NESSUS interface to ABAQUS are:

1. Any scalar quantity in ABAQUS can be treated as a random variable.
2. Geometry, pressure loading, and nodal temperature loading can be modeled using fully dependent random fields.
3. All ABAQUS procedures are valid.
4. Any ABAQUS response variable stored in the ABAQUS results file can be used for the probabilistic analysis or for inclusion in a life measure model.
5. Analysis options, random variable definitions, response identification, and life measure model are all selected from within the NESSUS input deck.
6. Automatic restart capability.
7. Batch mode operation.

These capabilities are described in more detail in the following section and by the example problem.

Random Variables

When performing probabilistic FE analysis, a realization of a random variable must be reflected in the FE input. A distinction is made between random variables that affect a single quantity in the FE input, called scalar variables, and random variables that affect multiple quantities, called field variables. Typical examples of scalar random variables include Young's modulus or a concentrated point load. Examples of field random variables are a pressure field acting on a set of elements or a geometric parameter that effects multiple node locations (e.g., radius of a hole).

In some situations, inputs to the deterministic FE model are a function of the random variables defined in the NESSUS input deck. For example, cross-sectional dimensions of a structural member such as width, depth, or thickness may be random variables. Rather than use these inputs directly, the deterministic FE code may alternatively require cross-sectional properties such as area and moments of inertia to be defined. To handle this and similar situations, NESSUS provides a Fortran subroutine for defining FE input quantities in terms of the input random variables. By using a programmable subroutine to define the relationship between FE input and the random variables, very general relationships can be defined.

Design Performance Measures

The performance measure for a given design is problem specific. FE analysis provides many results such as stress, strain and displacement to define performance. For nonlinear or transient problems, a measure of performance may be a plastic strain or acceleration. A key issue with combining probabilistic algorithms with FE analysis packages is extracting the results of interest. The approach used in NESSUS is to read analysis results for a given node or element and time step directly from the FE analysis package results file. General results selection options have been included in NESSUS when interfacing with FE analysis packages.

Sometimes the performance measure is a function of the FE results. For example, combining

the stress or strain results with fatigue equations to develop a distribution of life. In some instances the post-processing is a closed form equation or may require calling an external program to solve for the life measure. In NESSUS, a user-defined subroutine is provided to handle both scenarios. This subroutine provides a very general mechanism to combine finite element results with a life function.

Probabilistic Analysis Issues

Several practical issues arise when performing probabilistic analysis for complicated models. First, the analysis may terminate for unforeseen reasons such as power outages, system maintenance or lack of disk storage space. To prevent the loss of computational effort by starting at the beginning of the analysis, a restart option is available in NESSUS. The restart capability allows recovery from a computer crash by automatically starting at the point where the run terminated. The restart option is supported for all analysis procedures in NESSUS/PFEM: MV, AMV, AMV+, LHS, AIS, Monte Carlo simulation, and RSM. Once the FE results files have been created for a given set of random variable values, looping over results for multiple nodes and/or elements is very fast since the FE analysis is not rerun. The restart option also provides for improving a solution without rerunning any previous steps. For example, tightening the AMV+ convergence criteria or increasing the number of Monte Carlo samples would not require rerunning FE analyses associated with the previous steps of the methods.

Second a batch processing option was added to allow processing on different computers. This allows NESSUS to run on a local workstation while the finite element analysis program runs on a main frame or super computer. The batch processing option also allows for distributing the finite element analyses between different computers.

4. Example Problem

The example problem is a lever that is representative of an aircraft critical structural component. A deterministic analysis of a similar lever was completed for a future military aircraft using standard methods of mechanical analysis. The lever transfers load between an actuator and a control surface. The link must survive extreme, limit and normal operating loads without exceeding ultimate, yield or fatigue strengths, respectively. A

deterministic analysis may assume that all geometry variables are at the weakest extremes while loads are at their highest levels. The predicted stresses associated with the different load cases must be less than A-Basis material strengths, as defined in MIL-HDBK-5. This example investigates the fatigue failure mode.

Problem Description

The finite element model of the lever including the deterministic stress contours is shown in Figure 2. The model consists of 44844 degrees of freedom and each analysis required approximate 20 minutes on an HP 700 series workstation.

The random variables for the problem are listed in Table 1. The first two random variables effect the finite element model geometry. The hole location and radius random variables are shown in Figure 2. The tolerance for each dimension is assumed to be ± 0.01 inches and chosen to represent three standard deviations from the mean.

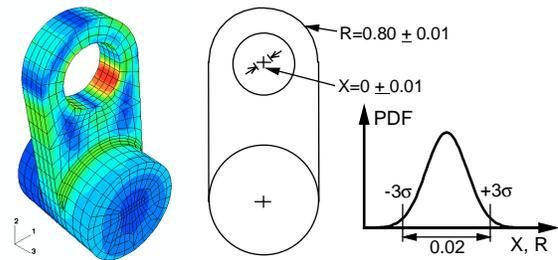


Figure 2. Finite Element Model, Stress Contours and Random Variables for Lever Example

Table 1. Random Variables for Lever Fatigue Example Problem

Name	Mean	Standard Deviation	Distribution Type
Hole Location	0.0	0.003333	Normal
Radius	0.8	0.003333	Normal
Operating Load	4.9	0.98	Normal
S-N Scatter	0	0.6	Normal

Because different realizations of these geometry random variables are required, a general approach is used to relate the finite element nodal coordinate input to a change in the random variable value. A vector, $\Delta \mathbf{x}$, is defined that relates how the coordinates change with a change in the random variable. The vector of perturbed nodal coordinates, $\hat{\mathbf{x}}$, is related to the mean value of the

coordinates, μ_x , plus a shift factor, s , times the amount of change for the coordinates, Δx , or in equation form,

$$\hat{x} = \mu_x + s \cdot \Delta x. \quad (3)$$

The shift factor is the difference between the mean value of the random variable and the perturbed value. One approach to generating Δx is to perturb the nominal mesh, subtract the nominal from the perturbed, and then normalize. This procedure is performed once and can then create a finite element mesh for any value of the random variable. This approach can be used for any type of field random variable (e.g., pressure and temperature distributions).

The load is assumed random and models the worst case tensile and compression loads through the range of motion of the lever. These loading extremes provide the stress range used for the fatigue life computations.

The example examines the distribution of fatigue life based on S-N data for AerMet100, shown in Figure 3. The equation for fatigue life was developed based on the data [8, 9]:

$$\text{Log}(N) = 26.4 - 9.16 \cdot \text{Log}(\sigma_{\max} (1-R)^{0.72}) + \psi \quad (4)$$

where N is the cycles to failures, σ_{\max} is the maximum principal stress, R is the ratio of the maximum and minimum stress and ψ is the scatter factor based on regression of the S-N data. Other techniques such as a prediction interval can be used to model the uncertainty of the S-N data [10].

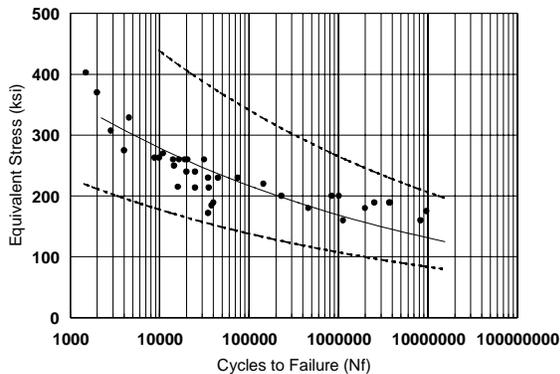


Figure 3. S-N Curve for AerMet100 with 3σ Bounds

Results

The probabilistic analysis was conducted for the fatigue life of the lever using the NESSUS interface to ABAQUS. Thirteen points of the cumulative distribution function (CDF) were computed using the AMV+ method and required 101 ABAQUS FE analyses (approximately 35 CPU hours on a 700 series HP workstation). The AMV+ CDF is shown in Figure 4 and is compared with 100 Monte Carlo samples. The Monte Carlo solution provides points on the CDF between approximately 0.01 and 0.99. For about the same computational effort, the AMV+ solution provides a CDF range of 10^{-5} to 0.999 and covers the lower or left tail probabilities where the designer is usually most interested. The AMV+ CDF compares well with these limited Monte Carlo samples. Based on AMV+ results, the probability is 99.994% that the fatigue life of the lever will be at least 1 million cycles.

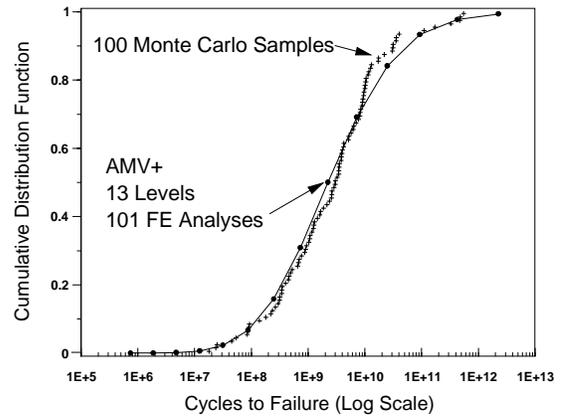


Figure 4. Cumulative Distribution Function

An important byproduct of the AMV+ method is the probabilistic sensitivity factors that identify the variables that contribute most to the reliability of the design (Figure 5). Other importance factors include the sensitivity of the p_f with respect to a change in the mean value or the standard deviation of each random variable. These normalized sensitivities are shown in Figure 6 and allow the designer to evaluate the effect of the probability of failure with a change of the design parameter. For this example, little importance is observed on the radius and hole location and these may be reviewed for manufacturing process changes that may lead to loosening of tolerances (cost reduction). The importance of the scatter on the fatigue life may influence the designer to invest in improved characterization of this variable or to obtain a cleaner material.

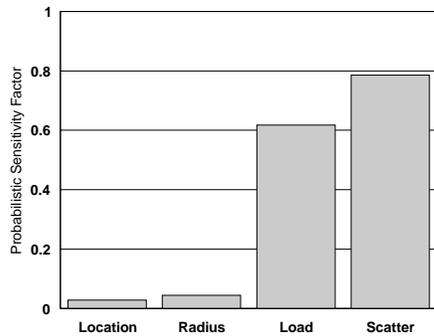


Figure 5. Probabilistic Sensitivity Factors

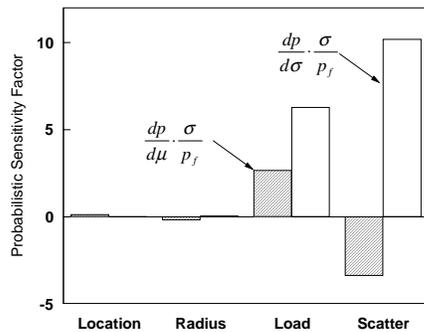


Figure 6. Sensitivity with Respect to Distribution Parameters

5. Conclusions

A probabilistic methodology and computational tool was developed and applied to compute the distribution of fatigue life for a general structure. The methodology combined the strengths of NESSUS for performing probabilistic analysis and ABAQUS for performing finite element analysis. The methodology was demonstrated on finite element model of a lever combined with a fatigue life model.

The probabilistic methods available in NESSUS were initially developed for aerospace applications; however, the methods are broadly applicable and their use warranted in situations where uncertainty is known or believed to have a significant impact on the structural response. With regard to the lever example represented here, the probabilistic results revealed several interesting aspects of the design/manufacturing parameters that would not have been apparent using conventional deterministic analysis methodologies.

Progress in probabilistic mechanics relies strongly on the development of validated

deterministic models, systematic data collection and synthesis to resolve probabilistic inputs, and identification and classification of failure modes. Future work in this area should include a more efficient means of computing perturbed finite element solutions. One approach would be a tighter link to commercial finite element analysis packages that have design sensitivity capabilities such as ABAQUS, NASTRAN and Pro/MECHANICA. This would require the commercial software developers to integrate the probabilistic capabilities of NESSUS directly into their software or provide links for more efficient communication with NESSUS.

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