UNCERTAINTY MODELING TO RELATE COMPONENT ASSEMBLY UNCERTAINTIES TO PHYSICS-BASED MODEL PARAMETERS

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Using physics-based models to predict the performance of engineered systems is becoming routine and is increasingly relied upon as a means to predict reliability when testing is prohibitive. To predict reliability, uncertainties in the system parameters must be modeled and propagated through the performance model using an appropriate probabilistic method. Uncertainties in engineered systems exist in loadings, environment, material strength, geometry, and manufacturing/assembly conditions. In many cases, these uncertainties are not direct physics-based model parameters. For example, the variations in the torque of a nut during assembly may be modeled as an initial penetration between two parts of the finite element model. Therefore, intermediate relationships between the physical uncertainties to the physics-based model are required. To account for these variations in the finite element model then requires a change to multiple nodal coordinates. Because of the significant time required to make these changes, a practical approach is required to model the geometry changes in complex finite element models. New capabilities in the NESSUS® probabilistic analysis software for creating and applying shape vectors to geometry changes have been developed and implemented and are described in the paper. The probability density functions used to model the uncertainties in these parameters are ideally developed using experimental data or expert judgment. This paper describes several uncertainty modeling approaches for an actual probabilistic analysis using a non-linear transient finite element model in excess of 1 million elements. Examples of combining computational models, analytical equations, and experimental results are presented to relate computational model inputs in terms of measurable random variables.

Nomenclature

D	=	distance travelled in a manufacturing process
δ	=	displacement
f	=	function
P	=	preload variable
x	=	random variable
σ	=	standard deviation

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ρ = correlation coefficient
ε = error term
b = intercept of a line
m = slope of a line
T = torque
F = preload
d_m = mean diameter of threads
λ = lead angle
μ = thread friction
2α = thread angle
μ_c = collar friction
d_c = mean collar diameter

I. Introduction

Physics-based models in the form of finite element and/or hydrocode analysis programs are used to predict the performance of engineered systems. Variations in the response can be predicted by propagating variations in the model parameters using an appropriate probabilistic method. Relating manufacturing parameters, material property variables, and environment variables to the computational model input may pose a challenge in performing a probabilistic analysis using a large-scale multi-physics model. In general, a probabilistic analysis of a complex system requires the decomposition of the equations and functions to their basic elements to allow each variable to be modeled accurately. This capability allows the response metric to be modeled using the observable or measured variables and not a derived quantity. The derived variable, for example, is an initial penetration in a finite element model and the observable variables are the friction coefficients and torque for a bolt tightening process. An additional benefit to this modeling approach is that probabilistic sensitivity factors are computed for the observable variables, some of which may be controlled to increase the design reliability.

Another challenge in performing the probabilistic analysis for large-scale models is to define how the finite element model changes when the portion of the geometry changes. The finite element model can be recreated from a solid model or finite element model preprocessing software. In many cases legacy finite element models are used where the solid model is no longer available. The NESSUS® geometric uncertainty modeling (GUM) tool was developed to graphically morph finite element geometry to create a shape vector that describes how the model changes with a change in a geometry parameter. Once this shape vector is created, the NESSUS probabilistic analysis software is used to recreate the finite element model for any value of the variable.[1] The NESSUS GUM capability provides a practical solution to model variations in finite element geometry.

Uncertainty models in the form of probability density functions are required to describe the inherent variations in the material properties, geometry, loads, and the manufacturing processes. The models for such inputs as torque and geometry can be determined from tolerance specifications. In other cases, this data is obtained from experiments such as for friction coefficients or other manufacturing parameters. For the case being investigated, a comparison of model predictions to experiments showed a large discrepancy and was highly influenced by an experimentally derived random variable. The investigation and derivation of this random variable using the combination of experimental results and computational models is described.

This paper describes several key elements of a probabilistic analysis that was performed to understand the importance of variations in assembly and material parameters in an engineered system. The performance of the engineered system was modeled using a non-linear transient finite element model in excess of 1 million elements. The assembly torque for several components in the system was modeled using an initial penetration between the nut and bolt components to achieve a specified preload. The preload was defined through torque and friction relationships. Uncertainties in the torque and friction between the components lead to variations in the initial penetration between the nut and bolt components. Approaches were developed to relate the torque and friction random variables to initial penetration.

The following sections describe several key modeling approaches used in a large-scale multi-physics probabilistic analysis. These models are then implemented as an example in the NESSUS software.

II. Uncertainty Models Developed from Computational Models and Experiments

A press fit process is used in a portion of the assembly of this system. A diagram of the shell and mount components is shown in Figure 1. A load is applied between 100 and 2600 lbs until a specific clearance is reached between the components in the system. The distance (D) traveled during this process is recorded and used to model variations in the simulation. Ten experiments were performed to estimate the mean and standard deviation of the distance and used as input to the finite element model. The preload variable was modeled as an applied displacement in the computational model.

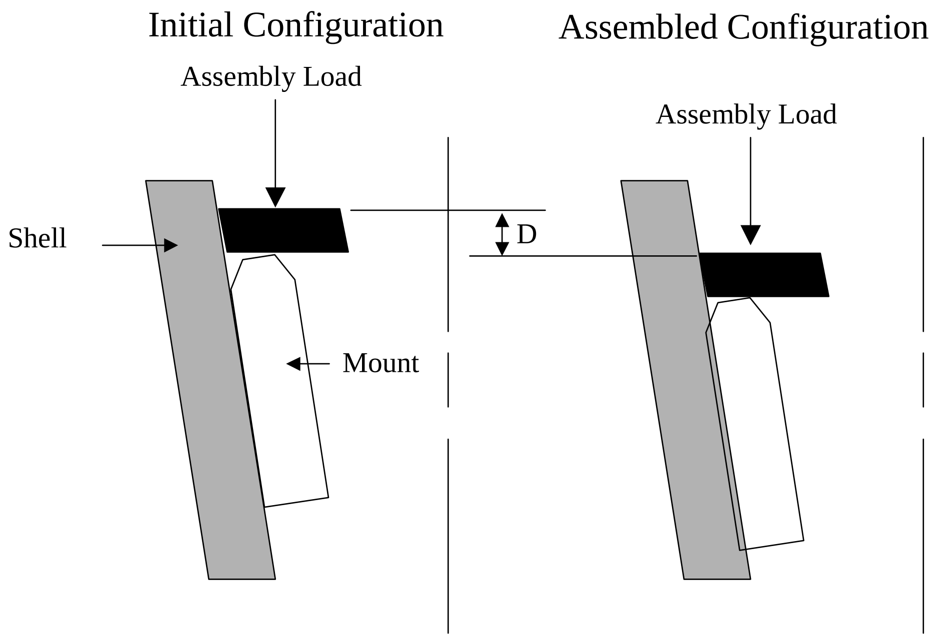


Figure 1. A load is applied to the mount and pressed until a specific clearance is reached. The distance (D) traveled during the process is recorded and used to model the variations in the assembly process.

The probabilistic model of this system included two torque loadings, friction coefficients, and environmental load. All random variables were used to predict the probabilistic response of a displacement metric at a specific location in the system. These displacements were then compared to the experimental results as shown by simulated data in Figure 2 (most data from this analysis is not available for public release and representative data is used). The much larger variations in displacement for the predicted results indicated a potential error in the model inputs. The following discussion describes the approach to understand what caused this increased variation.

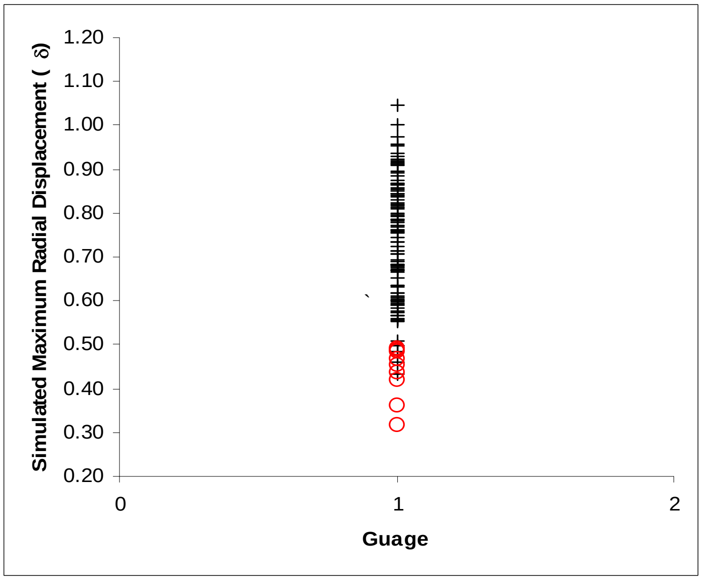


Figure 2. Simulated results representing predicted displacement (+) and those measured in the experiment (O).

The displacement metric (δ) can be represented as a function (f) of the preload variable (P) and other random variables (x_i) by

$$\delta = f(P, x_1, x_2, ..., x_n) \tag{1}$$

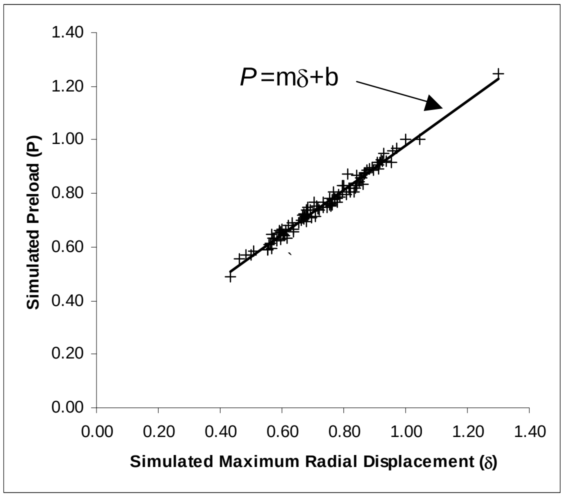


Figure 3. Functional relationship between displacement and preload variable using simulated data.

The displacement metric and preload variable demonstrated a high level of functional correlation using the computational model as shown in Figure 3. The displacement metric showed very little to no functional dependence to all other random variables.

The limited accuracy of the displacement measurements used to estimate the preload variable was determined to grossly increase the variation associated with this variable. The measured displacement metric during the experiment was expected to be much more accurate. An estimate of the standard deviation for the preload variable can computed from the measured displacement metric since the predictive model demonstrated that the displacement metric is only a function of the preload variable.

The variance for the displacement metric can be expanded as

$$\sigma_\delta^2 = \sigma_p^2 \left(\frac{\partial f}{\partial p}\right)^2 + \sigma_{x_1}^2 \left(\frac{\partial f}{\partial x_1}\right)^2 + ... + \sigma_{x_n}^2 \left(\frac{\partial f}{\partial x_n}\right)^2 + 2\rho\sigma_p\sigma_{x_1}\left(\frac{\partial f}{\partial x_1}\right)^2 + ... \tag{2}$$

By neglecting the random variables that have no influence on this displacement metric and ignoring higher order terms, the variance reduces to

$$\sigma_\delta^2 = \sigma_p^2 \left(\frac{\partial f}{\partial p}\right)^2 + \varepsilon \tag{3}$$

where ε is the error term associated with neglecting the higher order terms. Rearranging the equation for the variance of the preload variable yields

$$\sigma_p^2 = \frac{\sigma_\delta^2 - \varepsilon}{\left(\frac{\partial f}{\partial p}\right)^2}. \tag{4}$$

Using this formulation is conservative since including the higher-order terms associated with ε will reduce the variance for this variable.

A linear function can be developed between the displacement metric and the preload variable

$$f = \delta = \frac{P - b}{m} \tag{5}$$

with the terms defined in Figure 3.

The derivative of the displacement metric with respect to the preload variable is computed by taking the derivative of Eq. (5)

$$\frac{\partial f}{\partial p} = \frac{1}{m} \tag{6}$$

Substituting Eq. (6) into Eq. (4) and neglecting ε yields

$$\sigma_p^2 = \frac{\sigma_\delta^2}{\left(\frac{1}{m}\right)^2} \tag{7}$$

The standard deviation of the preload variable is thus

$$\sigma_p = m\sigma_\delta \tag{8}$$

These equations relate the standard deviation of the preload using relations developed from the computational model and experimental data.

III. Uncertainty Models for Torque Relationships

Part of the assembly process for this system is tightening a nut on a threaded component. The assembly specifications provide a specified torque with tolerances. The tightening of the nut is not modeled in the computational model but defined through an initial penetration between the two components. Several initial penetration depths were used to determine the relationship between the preload and initial penetration as depicted in Figure 4. This relationship defines the penetration depth as a function of preload

$$\delta = f(F) \tag{9}$$

δ is the initial penetration depth, *F* is the preload, and *f* is the function defined by the points in Figure 4.

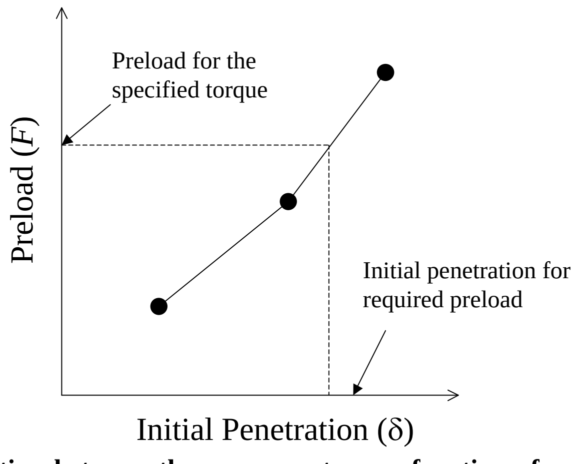


Figure 4. The initial penetration between the components as a function of preload is developed using finite element analysis and extracting the preload for several values of initial penetration.

The variations in the initial penetration are defined as a function of the uncertainties in the torque and two friction coefficients. The two contacting surfaces are those between the nut and collar and the nut and bolt. The relationship between the torque and preload, geometry, and friction coefficients is given by:[2]

$$T = \frac{Fd_m}{2}\left(\frac{\tan\lambda + \mu \ \sec\alpha}{1-\mu\tan\lambda\sec\alpha}\right) + \frac{F\mu_c d_c}{2} \tag{10}$$

$$\lambda = \tan^{-1}\left(\frac{l}{\pi d_m}\right) \tag{11}$$

Where

T – torque
F - preload
d_m – mean diameter of threads
λ – lead angle
μ – thread friction
2α – thread angle
μ_c – collar friction
d_c – mean collar diameter
l – thread lead

This torque equation is then inverted to determine the preload as a function of torque, geometry, and friction coefficients. This model can then be used to compute the initial penetration depth as a function of the torque and friction coefficient random variables.

IV. Geometric Uncertainty Modeling

When performing probabilistic finite element analysis, a specific (e.g., sample) realization of the random variables must be reflected in the finite element model input. Random variables that affect a single quantity in the finite element input are called scalar variables and random variables that affect multiple quantities are called field variables. Typical examples of scalar random variables include Young's modulus or a concentrated point load. Examples of field random variables are a pressure field acting on a set of elements or a geometric parameter that effects multiple node locations, e.g., radius of a hole.

Scalar random variables are directly mapped from the random variable value to the analysis program input. Field variables require a relationship between the random variable and the analysis program input. Because different realizations of these field random variables are required, a general approach can be used to relate the finite element input to a change in the random variable value. For example, if the random variable is the radius of a hole, changes to a set of nodal coordinate values will be required each time the radius is changed. This can be accomplished by defining a shape vector that relates how the finite element coordinates change for a given change in the random variable, i.e., radius in this example.

Figure 5 shows an example of a field random variable, where a change in the random variable *h* produces a change in the finite element mesh. This approach can be used for any type of field random variable (e.g., pressure and temperature distributions). If the scaling vector does not change during the analysis, then the relationship between the random variable and the finite element mesh is linear. Nonlinear relationships can also be defined if warranted.

A methodology has been proposed to facilitate the rapid mapping of a random variable to multiple finite element inputs allowing general transformations on the perturbation. Modeling geometric uncertainties in parameters such as gaps, dimensions, radii, etc. involve the movement of possibly thousands of nodes in large finite element models. The NESSUS Geometric Uncertainty Modeling (GUM) tool allows an existing finite element model to be easily parameterized. GUM is a graphical tool set for constructing shape vectors that define how the model changes for a specific variable, a process formerly requiring time-consuming setup. GUM facilitates the generation of the shape vector via the following capabilities:

- 3D visualization of finite element models via neutral file format converter
- Model simplification via material filtering
- Planar/Cartesian displacements
- Cylindrical displacements
- Model region selection by area, material, node location, and node ID query

The procedure for the shape vector creation involves

1. Graphically select nodes
2. Define transformation
3. Compute the delta vector
4. Verify displacement
5. Generate NESSUS input

The GUM tool is shown in Figure 6 where a cylindrical shape vector for changing the thickness of a thick cylinder (inner and outer radii) is developed.

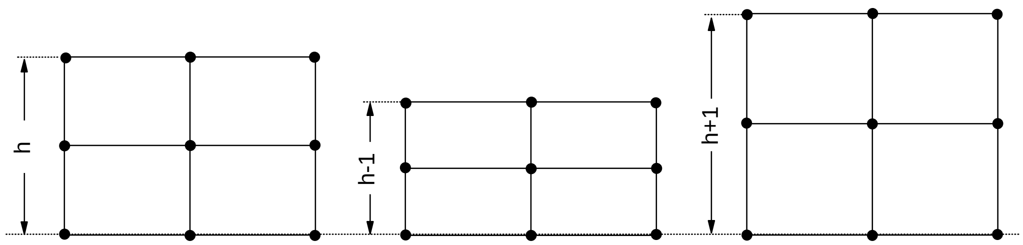


Figure 5. Describing how a random dimension, h, affects the FE mesh.

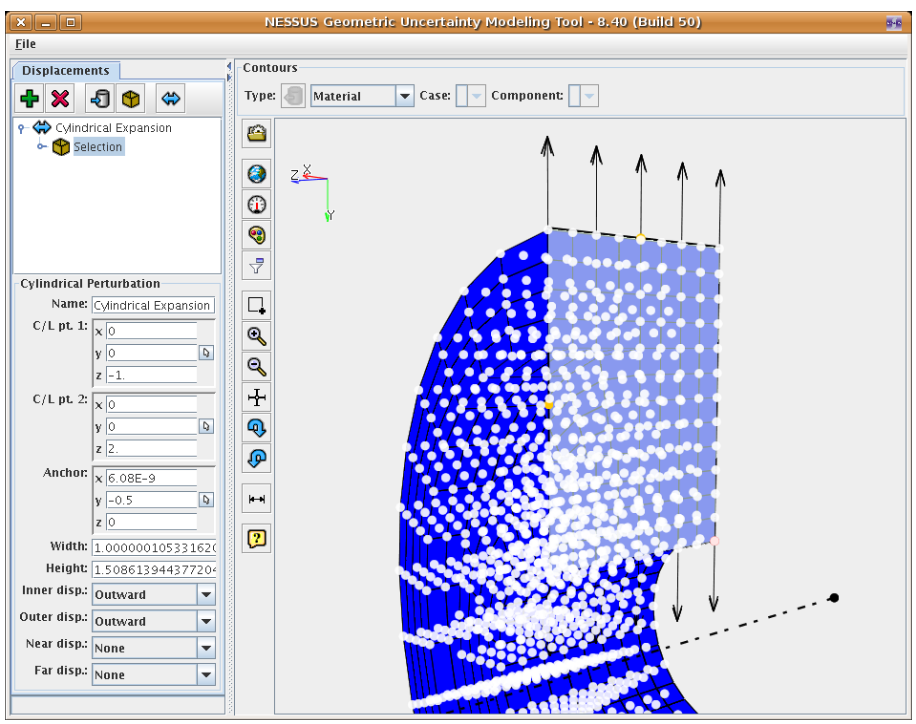


Figure 6. NESSUS Geometric Uncertainty Modeling (GUM) tool allows definition and visualization of shape changes in any finite element model due to variability in manufacturing, tolerances and assembly. Shape vectors are import directly into NESSUS for probabilistic analysis.

V. Example Problem

A simple but representative finite element model was developed to demonstrate the concepts outlined in the previous sections. Figure 7 shows the finite element model in the NESSUS geometric uncertainty modeling tool. The red component represents the mount and the blue component is the shell in Figure 1. The dashed lines with letters at the vertices identify the region of the model that will be scaled. The vertices can be moved independently to represent complex transformations and scaling. The translated dashed shape is a representation of the transformation based on the created shape vector. This shape vector relates the preload variable to an initial penetration in the finite element model by translating the specified nodes.

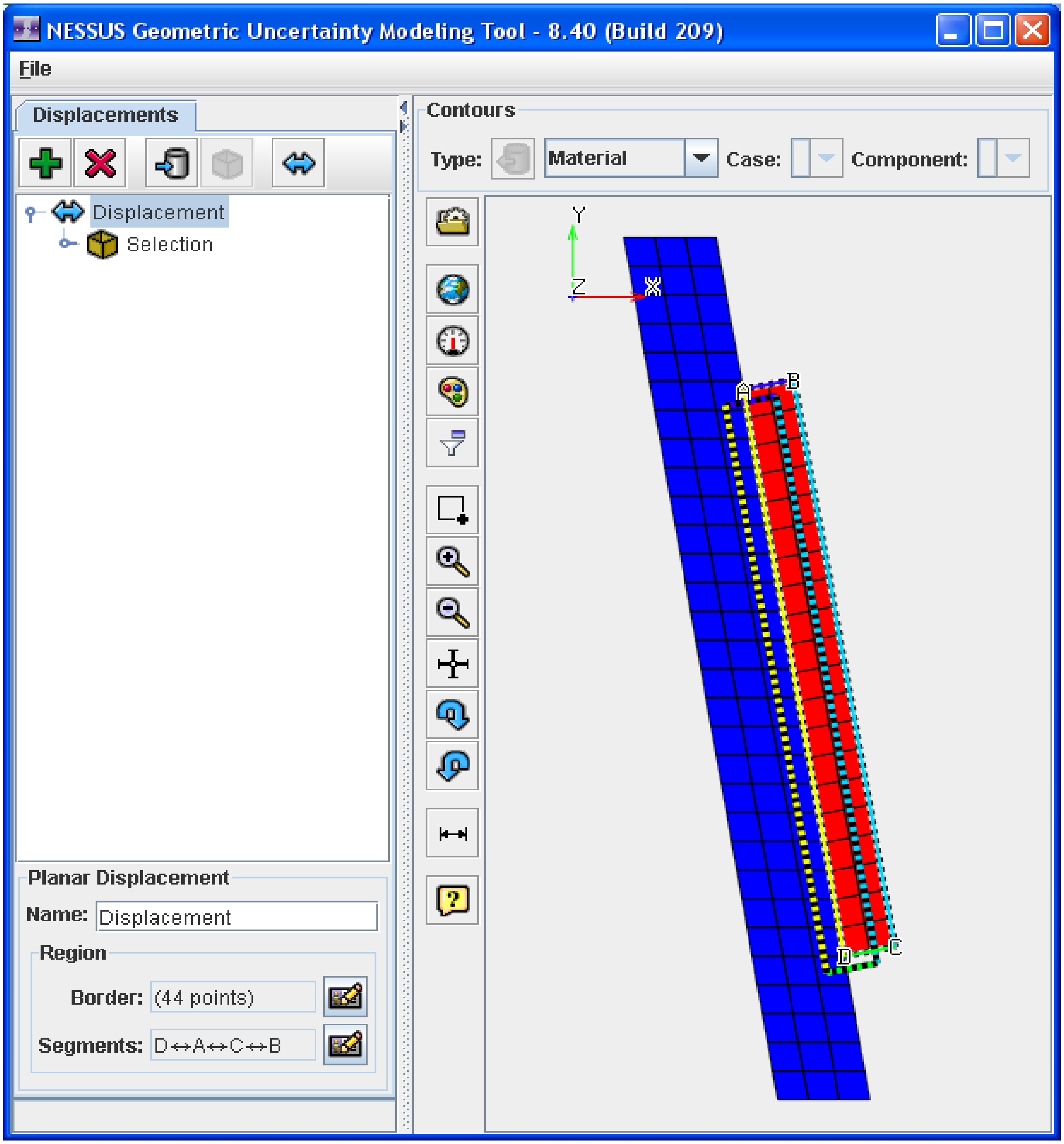


Figure 7. NESSUS geometric uncertainty modeling tool is used to create a shape vector to represent the initial penetration of the red component.

A sample problem was defined in the NESSUS software to further describe the modeling of the problem. The example relates a torque to an initial penetration. The problem statement is shown in Figure 8 and is evaluated from the bottom to the top. The first two equations (from the bottom) describe the torque and preload relationships from Eqs. 10 and 11. The next equation defines the initial penetration as a function of preload (Eq. 9 and Figure 4). The top level function relates the response of interest to a function FE. The FE function can be defined from a library of interfaces to commercially available finite element codes, in-house tools, and/or user-defined external programs. This function has arguments that include both deterministic and random variables that are used in the finite element analysis. The delta variable describes the initial penetration in a threaded component. A shape vector for this initial penetration can be developed using the NESSUS GUM tool. The mu_t and mu_c are friction coefficients that are using in the preload/torque relations and also in contact definitions in the finite element model. The x1 and x2 variables represent other load and material variables.

The preload variable models the initial penetration described in Figure 1 with the shape vector defined by the process shown in Figure 7. This shape vector is mapped to the finite element input file in NESSUS. NESSUS provides a graphical tool to highlight the rows and columns that will change when this variable changes. The mapping screen for this variable is shown in Figure 9. This mapping capability provides a quick and accurate means to relate variables to the finite element input. The graphical approach allows visualization to verify that variables are mapped to correct portions of the finite element input. A finite element model can be automatically created for any value of the variables once the shape vectors are developed and variables are mapped to the finite element input.

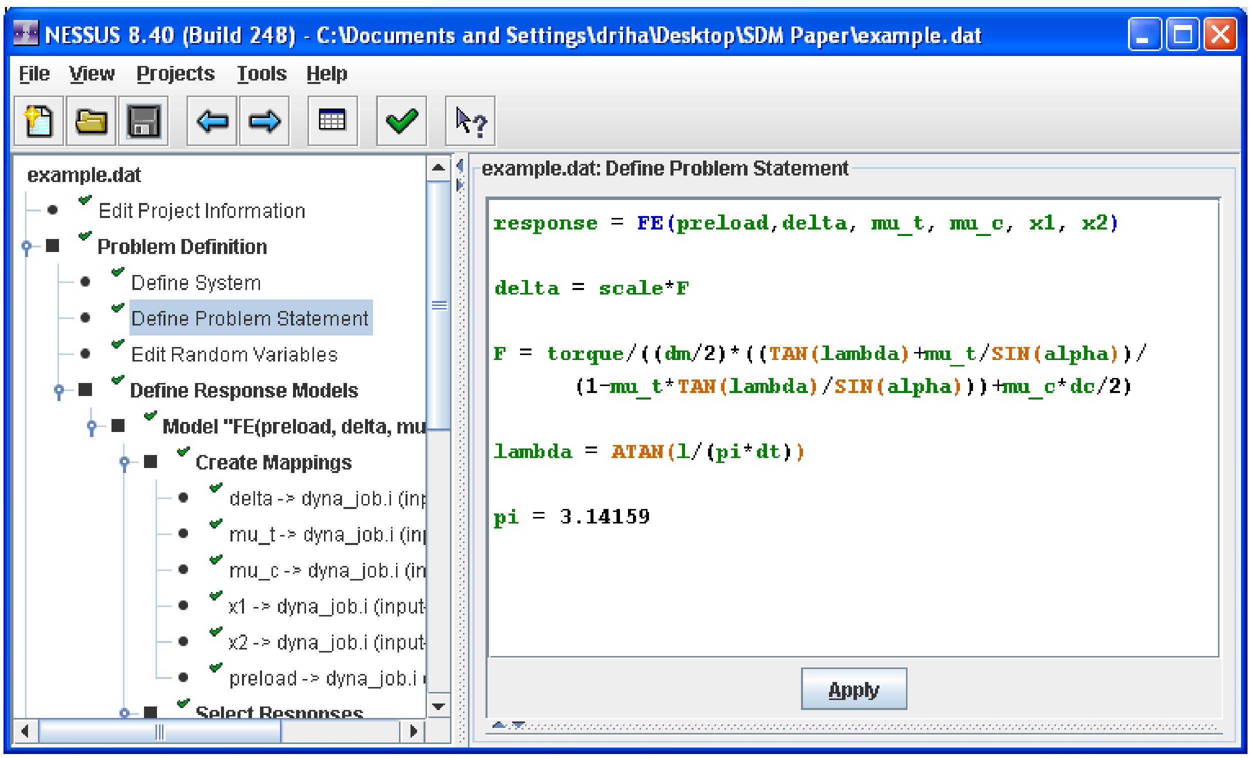


Figure 8. NESSUS problem statement relating torque to the initial penetration in a finite element model.

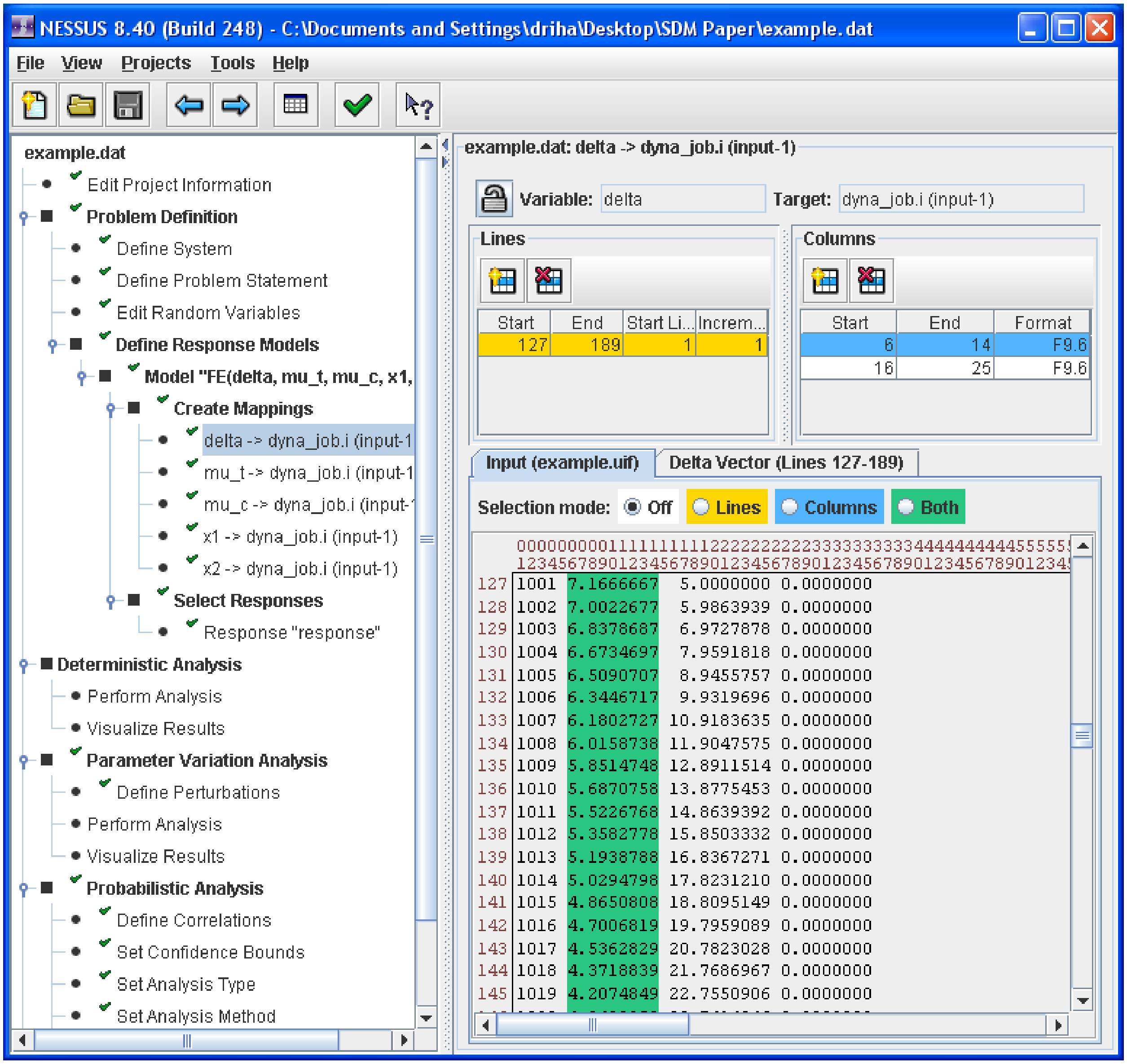
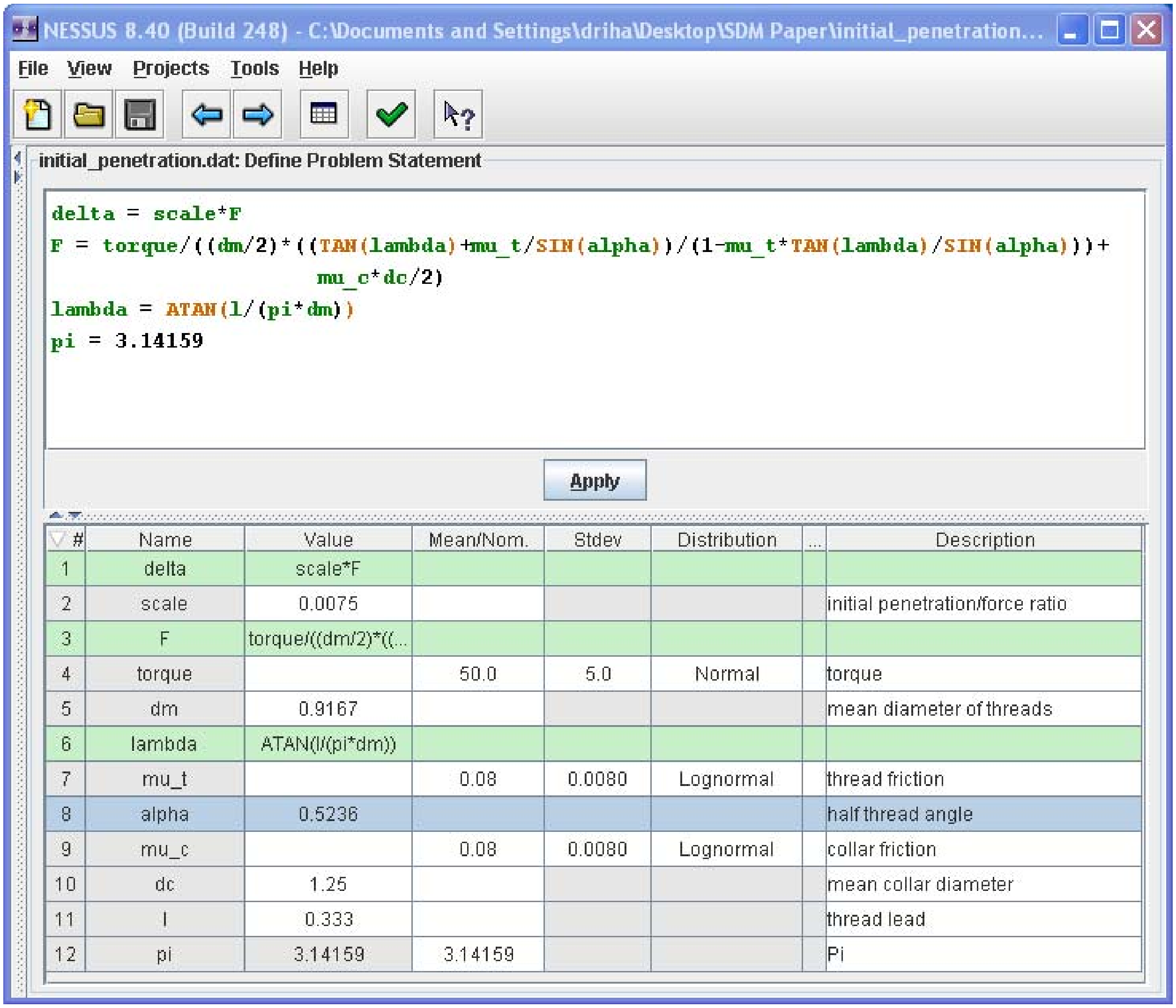


Figure 9. The delta vector is mapped to the finite element model for the preload variable.

An example probabilistic analysis was performed to compute the probabilistic response of the initial penetration using the torque/penetration relationships and assumed input distributions. The NESSUS problem statement and variable definitions are shown in Figure 10. The first order reliability method (FORM) in NESSUS was used to compute the cumulative distribution function of the initial penetration for this example (Figure 11). The probability that the penetration is greater then 0.3 is 0.0002. This probability can be read from the CDF in Figure 11 or computed directly by NESSUS.



#	Name	Value	Mean/Nom.	Stdev	Distribution	...	Description
1	delta	scale*F					
2	scale	0.0075					initial penetration/force ratio
3	F	torque/((dm/2)*((...					
4	torque		50.0	5.0	Normal		torque
5	dm	0.9167					mean diameter of threads
6	lambda	ATAN(l/(pi*dm))					
7	mu_t		0.08	0.0080	Lognormal		thread friction
8	alpha	0.5236					half thread angle
9	mu_c		0.08	0.0080	Lognormal		collar friction
10	dc	1.25					mean collar diameter
11	l	0.333					thread lead
12	pi	3.14159	3.14159				Pi

Figure 10. NESSUS problem statement for example probabilistic analysis.

The probabilistic analysis also provides probabilistic importance and sensitivity values, which identify the variables that contribute most to variations in the initial penetration. The probabilistic importance factors are shown in Figure 12 and provide a relative ranking of the importance of each variable. These importance factors identify torque as the most important variable for exceeding an initial penetration of 0.3.

The probabilistic sensitivities for this case are shown in Figure 13. These sensitivities are derivatives of the probability of exceeding a specified penetration with respect to the mean and standard deviation of each random variable. This case shows that the probability of exceeding an initial penetration of 0.3 can be decreased most by reducing the standard deviation (tightening the tolerance) of the torque. The sensitivities also show that the probability can be decreased if the torque mean is decreased. The sensitivities for the friction coefficients have the opposite sign and indicate the probability of failure will increase if the mean friction decreases.

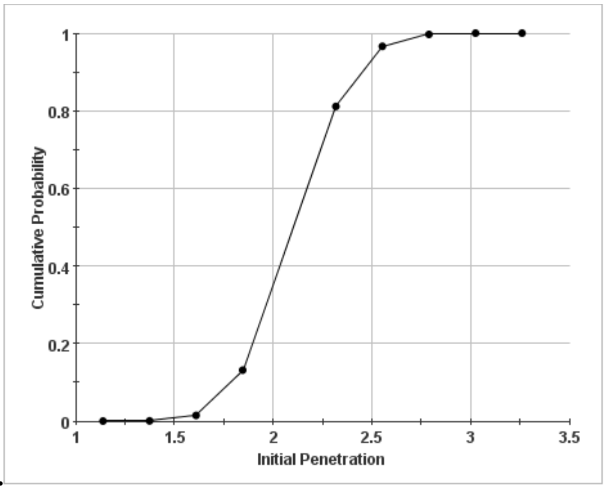


Figure 11. Cumulative distribution function for initial penetration in the example problem.

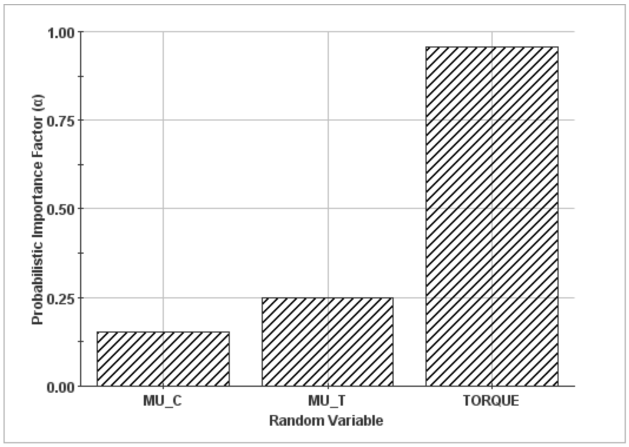


Figure 12. Probabilistic importance factors for example problem

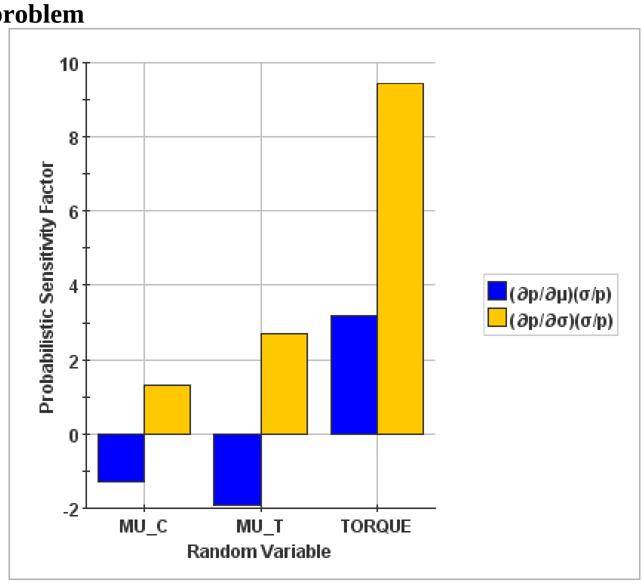


Figure 13. Probabilistic sensitivity factors for example problem

VI. Summary

A detailed probabilistic analysis was performed using a large-scale multi-physics based model to predict the probabilistic response of the performance metrics and the probabilistic sensitivity measures. Several modeling approaches were described to relate manufacturing variations to computational model inputs. The key concepts for these modeling approaches were then demonstrated using the NESSUS probabilistic analysis software.

Performing a probabilistic analysis using a large-scale multi-physics based model may require describing the input variables in terms of the measurable or observable variables and not always only in terms of the inputs to the computational model. The modeling process described in this paper defined a relationship between initial penetration (to model a preload in the computational model) and torque and friction (both measurable quantities). The computation model was used to determine the relationship between preload for the torque relations and initial penetration. The next step was to define how the finite element model geometry changes for an initial penetration value. The NESSUS geometric uncertainty modeling tool was used to define a shape vector for the initial penetration. Once this shape vector was defined, a computation model input file can be created for any value of initial penetration.

In general, a probabilistic analysis of a complex system requires:

- The decomposition of the equations and functions to their basic elements allows the response metric to be modeled using the observable or measured variables and not a derived quantity. This approach provides probabilistic sensitivity factors for the observable variables, some of which may be controlled to increase the reliability,
- The combination of computation models, analytical equations, and experimental results to develop the sub-models to accurately model the measureable random variables and how these variables change the basic computation model inputs, and
- An approach to develop relationships between the random variable and the finite element model inputs, especially for field type variables. The NESSUS GUM capability addresses this issue by providing a graphical tool to morph complex geometries.

Acknowledgments

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References

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[2]Shigley, J . E.; and Mitchell, L. D., *Mechanical Engineering Design*, Fourth Edition. McGraw-Hill Inc., 1986.