



Modeling Gas Turbine Engine Performance at Part-Load

Author

Patrick Thomas Weber

Supervisor

Dale Grace

May 2011 - July 2011

Submitted in partial fulfillment of the requirements of the
UTSR Gas Turbine Industrial Fellowship Program

Electric Power Research Institute
Southwest Research Institute
University of Wyoming

1 Introduction

Gas turbine engines are appealing for ground based power production because of their low cost-of-entry, relatively high thermal efficiency, and flexibility in output power. Gas turbines are typically used in peaking power applications, where base load power will not meet the current grid demand. Also, gas turbines' intrinsic high thermal efficiencies in combined cycle applications are being used to provide base load power as well.

Also, gas turbines are attractive because of their ability to quickly ramp up power production. Where coal fired, steam cycle power plants can take anywhere from hours to days to produce full-load power, a gas turbine power plant can move from cold start to full power in a matter of minutes [1].

As compared with a nuclear or coal fired power plant, the fixed cost of a gas turbine engine plant is relatively low. According to a study performed by the Department of Energy, advanced combined cycle gas turbine engine configurations fired by natural gas have the lowest levelized cost when compared with coal, wind, solar, nuclear, and other plant types [2]. Gas turbine marginal costs are typically higher as they are firing a more expensive and energy rich fuel. However, as they are typically used in conditions where peaking power is required, this incremental cost is typically offset by their lower fixed cost.

When a gas turbine engine power plant is used for peaking power, engine output power can vary depending on grid demand. However, manufacturers typically only specify engine performance characteristics at full-load. In this case, part-load and other off-design operating conditions are undefined.

The objective of this project was to reverse engineer full-load thermodynamic characteristics for modern gas turbine engines used for ground based power production and then derive part-load performance thermodynamic characteristics.

2 Gas Turbine Engine Modeling

Other gas turbine engine part-load models have used a Second-Law of Thermodynamics (accounting for the irreversibility of a process) based approach [3] [4]. While this approach can be useful for determining where losses and other such exergy and entropy generation and destruction is occurring, an energy based approach was chosen instead for its accuracy.

2.1 Thermodynamics

For the design of the part-load models, the gas turbine is assumed to be in a single shaft, simple cycle configuration as shown in Figure 1.

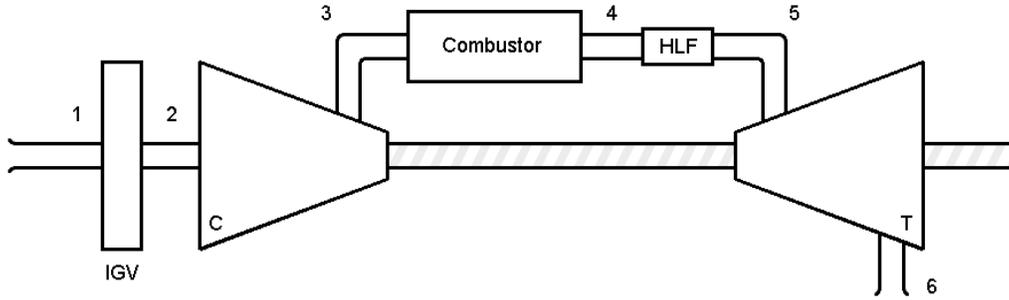


Figure 1: Single Shaft, Simple Cycle Configuration

All modeling is entirely thermodynamic in its nature and does not take into account flow swirl, geometry, or other advanced computational fluid dynamics. All state parameters such as temperature, pressure, and mass flow rate still must be solved.

Table 1: Simple Cycle Gas Turbine State Parameters

1	Ambient Conditions Pre-IGV
2	Post-IGV Pre-Compressor
3	Post-Compressor Pre-Combustor
4	Post-Combustor Pre-HLF
5	Post-HLF Pre-Turbine
6	Post-Turbine Exhaust Gas

Based on Figure 1 and Table 1 with six states, more than 18 operating parameters must be calculated. In actuality, the number of parameters is much higher after other parameters such as compressor and turbine efficiency, cycle efficiency, and fuel heating value, for example, are introduced.

2.2 Core Equations

The thermodynamic engineering models revolve primarily around the First Law of Thermodynamics. Energy through the engine must be conserved. The energy balance equation is:

$$m_{exhaust} (C_{P,5}T_5 - C_{P,6}T_6) = W_{output} + m_{air} (C_{P,3}T_3 - C_{P,2}T_2) \quad (1)$$

However, manufacturer parameters did not assure conservation of energy. Therefore, the models were adjusted accordingly. The inconsistency with the manufacturer parameters has led to the creation of an intermediate state, between the combustor outlet and the turbine inlet, defined as the ‘‘Heat Loss Factor,’’ or HLF.

Since the process from State 4 to State 5 is not at a constant volume, the pressure change associated with the temperature change is non-existent.

$$T_5 = T_4(1 - \overline{HLF}) \quad (2)$$

$$P_5 = P_4 \quad (3)$$

The function of a compressor is to impart kinetic energy into the flow stream as it enters the compressor, and then to convert its kinetic energy to pressure. The function of a turbine is to convert energy (both heat and kinetic) into useful shaft work. The governing equations, including the effects of a non-perfect compressor and turbine (efficiency) is accounted for as shown in Equation 4 and Equation 5.

$$T_3 = T_2 \left(\frac{P_3}{P_2} \right)^{\frac{\gamma-1}{\gamma} \frac{1}{\eta_{comp}}} \quad (4)$$

$$T_6 = T_5 \left(\frac{P_6}{P_5} \right)^{\frac{\gamma-1}{\gamma} \eta_{turb}} \quad (5)$$

Note that the pressure fraction in the compressor equation is equal to the engine’s pressure ratio.

$$PR = \frac{P_3}{P_2} \quad (6)$$

Further, to account for the combustor in the set of pressure equations, a ‘‘combustor pressure drop’’ term was included. This is assumed to be 4% on average, but can also be redefined on an engine-by-engine basis.

$$P_4 = P_3(1 - \Delta P_{combustor}) \quad (7)$$

To account for the change in temperature across the combustor, a simple relationship between the fuel mass flow rate and the change in temperature is used.

$$m_{fuel}LHV = m_{exhaust}C_{P,4}T_4 - m_{air}C_{P,3}T_3 \quad (8)$$

The exhaust gas flow rate is simply the sum of the fuel mass flow rate and the air mass flow rate.

$$m_{exhaust} = m_{air} + m_{fuel} \quad (9)$$

Also, of particular interest is the overall thermal cycle efficiency. This term can also be considered a non-dimensionalized *heat rate*. This parameter is provided at full-load operation by the manufacturers.

$$\eta_{cycle} = \frac{W_{output}}{Q_{input}} = \frac{W_{output}}{m_{exhaust}C_{P,4}T_4 - m_{air}C_{P,3}T_3} \quad (10)$$

Finally, for the off-design and part-load cases, two assumptions are made for the engine. First, the flow into the turbine is assumed to be choked, meaning the flow is travelling at or near transonic conditions. After simplification, the equation looks like:

$$m_{total} \frac{\sqrt{T_5}}{P_5} = m_{total,full} \frac{\sqrt{T_{5,full}}}{P_{5,full}} \quad (11)$$

Second, the other assumption being made is that of constant, or prescribed volumetric flow rate across the compressor. Depending on the location in the control curves, the volumetric flow rate multiplier (VFR) in front of the $m_{air,full}$ term in Equation 12 will scale.

$$m_{air} = vfr \cdot m_{air,full} \frac{T_2 P_2}{T_2 P_{2f}} \quad (12)$$

Accordingly, the pressure drop term across the inlet guide vane, P_2 , is assumed not to cause any noticeable temperature change. Attempting to model the temperature change resulted in a total ΔT of 0.2°R over the entire load range.

$$T_2 = T_1 \quad (13)$$

It becomes apparent through the comparisons between the number of equations and the number of unknowns, one equation is missing in order to completely define the solution set. Normally, a singular solution to this problem would not exist, but this missing equation is what allows the

engine to be controlled. More discussion about engine control can be found in Section 2.4.

2.3 Model Types

Once the core equations are defined, the full-load ISO, full-load off-design, and part-load models can all utilize the same equations. With the exception of the full-load ISO model, all models use a reduced form of 11 equations, although with different assumptions. The varying assumptions are discussed further in Sections 2.3.1, 2.3.2, and 2.3.3.

2.3.1 Full-Load Model

The full-load model utilizes the manufacturer input parameters [8], and using the equations listed previously, calculates the temperature, pressure, and mass flow rate at every state. Making an assumption about the turbine efficiency also allows the turbine rotor inlet temperature and the compressor efficiency to be calculated.

A primary difference between the full-load ISO model and the other model types is that the full-load model does not assume constant (or prescribed) volumetric flow rate across the compressor, nor does it assume choked flow at the turbine inlet. This is apparent, simply because those equations would evaluate to an equality (i.e. $1 == 1$) at full-load.

2.3.2 Off-Design Model

The off-design (or full-load off-design) model uses the parameters from the ISO full-load model, with modifications to the input parameters. These modifications include ambient pressure (elevation) and temperature changes, as well as exhaust pressure modifications (i.e. backpressure).

Rather than assume a known work output or pressure ratio, these terms are allowed to vary, along with exhaust gas mass flow rate. The compressor efficiency is assumed to follow a compressor efficiency curve, and the turbine rotor inlet temperature is held at the same value as it would be at ISO conditions.

This allows all parameters to be calculated at an off-design condition through the use of the state values at ISO conditions.

2.3.3 Part-Load Model

The part-load model sparingly uses the parameters from the full-load model at off-design condition (or ISO if no off-design case is used). The only assumed constants are the ambient conditions,

exhaust pressure, heat loss factor, the turbine and compressor efficiencies (scaled for off-design conditions), and the percent of full-load power output.

The only issue is that with the part-load case, there is one extra unknown. Under any normal mathematical solution technique, this would be a problem. However, this is actually a deliberate feature in the model which allows the engine to be controlled in the solution method.

2.4 Engine Control Theory

In order to maintain synchronous speeds with the gas turbine while still reducing or increasing power output, the engine must be properly controlled. The control of the engine depends on where in the part-load curve the engine is operating.

Based on the analysis of manufacturer curves from GE, Alstom, Siemens, etc., two different control techniques were implemented, each with three separate stages of control. The method of control varies from engine to engine and from manufacturer to manufacturer, but they primarily consist of two requirements. Either a constant exhaust temperature is desired, or a constant turbine inlet temperature is desired. Both control methods will be discussed in detail, but a generalized figure can be found in Figure 2.

For either control case, the engine control computer maintains a constant mass flow rate as power output is reduced from full-load. In order to reduce the power output, turbine inlet temperature is reduced, which in turn, reduces the exhaust temperature.

Turbine rotor inlet temperature is allowed to fall until it hits a pre-set minimum value. In the case of the example curve shown in Figure 2, the minimum turbine rotor inlet temperature is 95% of full-load turbine rotor inlet temperature. This control technique is primarily used to enhance the lifetime of the hot-section components and to reduce the number of maintenance operations on the gas turbine engine. At this point, the inlet guide vane modulation is activated and air mass flow rate is no longer assumed to be constant, but rather a function of the other state parameters.

The act of modulating the inlet guide vanes (or variable stator vanes, depending on the engine) induces a pressure drop across the inlet guide vanes. This pressure drop reduces compressor inlet air density thereby reducing the overall air mass flow rate. It is also at this point where the two engine control techniques deviate, so each control technique will be discussed in their own section.

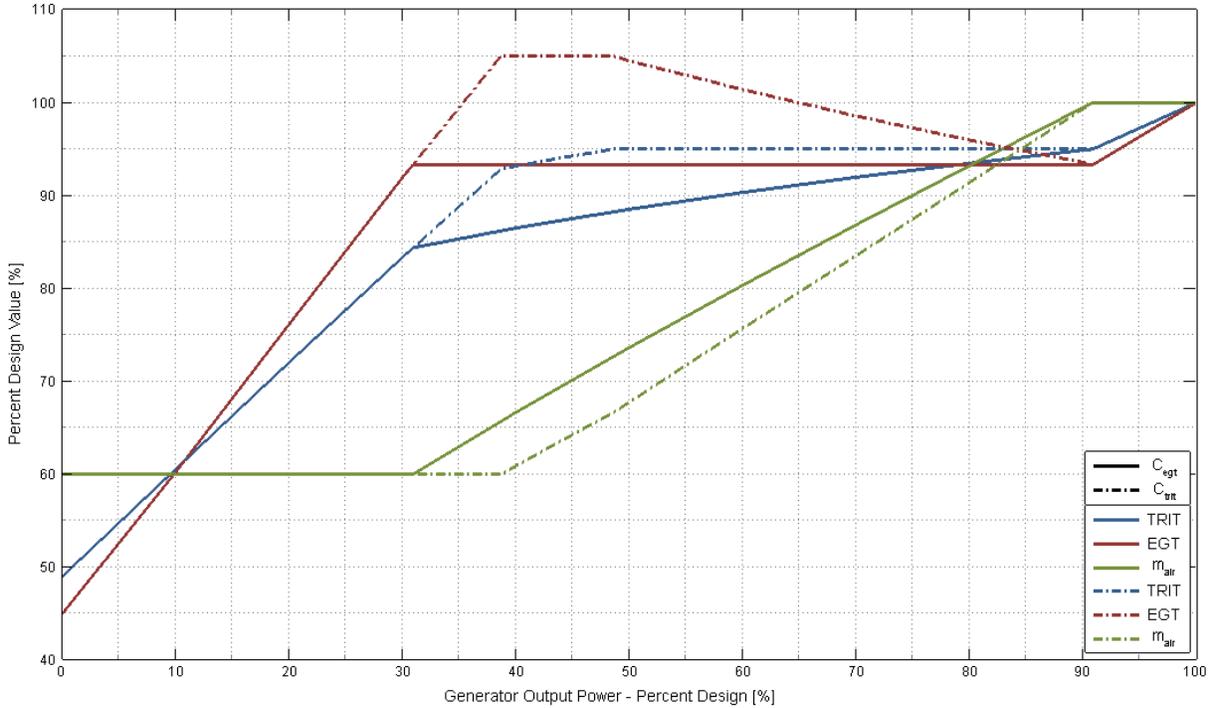


Figure 2: Engine Control Curve Impact on TRIT, EGT, M_{air}

2.4.1 Constant Exhaust Gas Temperature

In the case that exhaust gas temperature (EGT) is to remain constant, the control computer will both modulate turbine rotor inlet temperature (TRIT) and air mass flow rate (AMFR) to ensure this condition is met. For the control curve shown in Figure 2, the constant exhaust gas temperature control curve is shown as the red, green, and blue solid lines.

The exhaust gas temperature is held constant, while turbine rotor inlet temperature and air mass flow rate decrease. Since the air mass flow rate is primarily controlled by the inlet guide vane modulation, there exists some minimum value at which the inlet guide vanes become fully closed and no longer throttle the inlet air pressure. For the curve shown in Figure 2, this point was defined to be 60% of full-load air mass flow rate.

At the 60% load point, the inlet guide vanes are fully closed and air mass flow rate can no longer be modulated. This requires a decrease in turbine rotor inlet temperature (through a drop in fuel mass flow rate), which also causes a corresponding decrease in exhaust gas temperature. Both are allowed to fall until a full speed, no load condition (3600 RPM, 60 Hz) is met.

2.4.2 Constant Turbine Rotor Inlet Temperature

In the case that turbine rotor inlet temperature (TRIT) is to remain constant, the control computer will modulate air mass flow rate to ensure this condition is met. For the control curve shown in Figure 2, the constant turbine rotor inlet temperature curve is shown as the red, green, and blue dashed lines.

When the turbine rotor inlet temperature is held constant, the exhaust gas temperature increases and the air mass flow rate decreases. As air mass flow rate decreases, exhaust gas temperature increases to maintain energy balance. However, due to hardware limitations with both the gas turbine hardware and the steam cycle hardware used in the combined cycle configuration, the exhaust gas temperature is only allowed to rise until a maximum value is reached. In the case of the control curve shown in Figure 2, a maximum exhaust gas temperature of 105% of full-load exhaust gas temperature was assumed.

At the point that the exhaust gas temperature reaches the maximum value, the control computer can no longer maintain a constant turbine rotor inlet temperature while still reducing power output. In order to reduce power output while preventing the exhaust gas temperature to rise above its maximum, turbine rotor inlet temperature must be decreased. The exhaust gas temperature now becomes the control point.

Turbine rotor inlet temperature is then decreased continuously until the air mass flow rate of the engine reaches a minimum value. It is at this point that the inlet guide vanes enter a fully closed condition and can no longer throttle inlet air pressure.

At this minimum air mass flow rate, the inlet guide vanes are fully closed and air mass flow rate can no longer be modulated. This requires a decrease in turbine rotor inlet temperature (through a drop in fuel mass flow rate), which also causes a corresponding decrease in exhaust gas temperature. Both are allowed to fall until a full speed, no load condition (3600 RPM, 60 Hz) is met.

2.5 Numerical Solver

Even after simplification, the complexity of the equations and their inherent non-linearity means that they cannot be solved algebraically. Attempts at solutions with Maple, MATLAB, and Mathematica confirm this. In order to solve these equations, a numerical method was selected.

The Newton-Raphson method, or simply Newton's method, is a root finding algorithm that readily lends itself to solving single equations, and systems of equations. Its inherent mathematical simplicity also allows it to solve systems of non-linear equations with the knowledge only of a single root. Unfortunately, there are a few cases where the method will fail to converge making Newton's method not as robust as other methods.

Newton's method approximates successive roots by relating the function to its first derivative.

Assuming the derivative exists and is real, especially near the root, the function will converge quadratically, meaning the number of accurate decimal points doubles with each iteration. This means that Newton's method converges very quickly for well-behaved equations.

The basic mathematical equation describing Newton's method is shown in Equation 14.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (14)$$

In order to convert Newton's method into a form that can be used for solving simultaneous systems of non-linear equations, Equation 14 was converted into a vector-matrix form. The vector-matrix form allows the system to support multiple equations over multiple solution domains. This vector-matrix form is shown in Equation 15.

$$\mathbf{X}_{n+1} = \mathbf{X}_n - \frac{\mathbf{F}(x_n)}{\mathbf{F}'(x_n)} \quad (15)$$

The problem with this equation comes from the evaluation of the derivative of a vector and its successive vector/vector division, which is not a construct known in linear algebra. The evaluation of a partial derivative with each unknown in the equation simply means the vector derivative becomes a Jacobian matrix, as shown in equation 19.

$$\mathbf{J} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \dots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \dots & \frac{\partial y_m}{\partial x_n} \end{bmatrix} \quad (16)$$

However, in linear algebra, the inverse of the Jacobian matrix is used, leading to the final form of the equation as shown in Equation 17.

$$\mathbf{X}_{n+1} = \mathbf{X}_n - \mathbf{J}^{-1}\mathbf{F}(x_n) \quad (17)$$

In order to solve for the inverse Jacobian, the Jacobian is first numerically approximated through the use of a *for* loop. Since the vertical columns of the matrix are all with respect to one variable, they can all be calculated simultaneously by perturbing the first variable, and reevaluating all functions. This reduces the computation time since, rather than nesting two *for* loops to calculate each element in the Jacobian (121 elements for an 11x11 matrix), each column is calculated at each iteration, reducing the loop count down to n iterations (where n is simply the length of one side of the square Jacobian matrix, i.e. $n = 11$ for an 11x11 matrix).

Then, this Jacobian matrix, once complete, is inverted using the Gauss-Jordan method for solving

linear algebraic equations (matrix), which is quick and robust.

2.6 Gauss Jordan Method

In order to solve the inverse of the Jacobian, the Gauss-Jordan method of matrix inversion was used. Basically, the method works on the principle of element row operations.

First, the Jacobian matrix is augmented by appending an identity matrix to the original Jacobian matrix, making it a $2nxn$ matrix.

$$\mathbf{J} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_n} & 1 & 0 & 0 \\ \vdots & \ddots & \vdots & 0 & 1 & 0 \\ \frac{\partial y_m}{\partial x_1} & \cdots & \frac{\partial y_m}{\partial x_n} & 0 & 0 & 1 \end{bmatrix} \quad (18)$$

Then, elementary row operations such as matrix row switching, multiplication, and addition are performed on the matrix until the left hand side of the Jacobian matrix is divided out to be equal to the identity matrix. Effectively, the matrix is flip-flopped exchanging the original matrix with the inverted matrix.

This technique leaves the Jacobian on the right hand side of the augmented matrix with an identity matrix on the left.

$$\mathbf{J}^{-1} = \begin{bmatrix} 1 & 0 & 0 & J_{0,0}^{-1} & \cdots & J_{n,0}^{-1} \\ 0 & 1 & 0 & \vdots & \ddots & \vdots \\ 0 & 0 & 1 & J_{0,n}^{-1} & \cdots & J_{n,n}^{-1} \end{bmatrix} \quad (19)$$

This method requires $O(n^3)$ operations, so it is both relatively computationally efficient (as compared to methods requiring $O(n!)$ computations).

2.6.1 Logic Branching

In order to solve the system while including the methods for engine control, a new way of stepping through the models had to be created. Logic Branching, as it has been called, is simply a way to determine where, within the model, the part-load model is asked to be solved, and how the program must step through the model to determine the appropriate result.

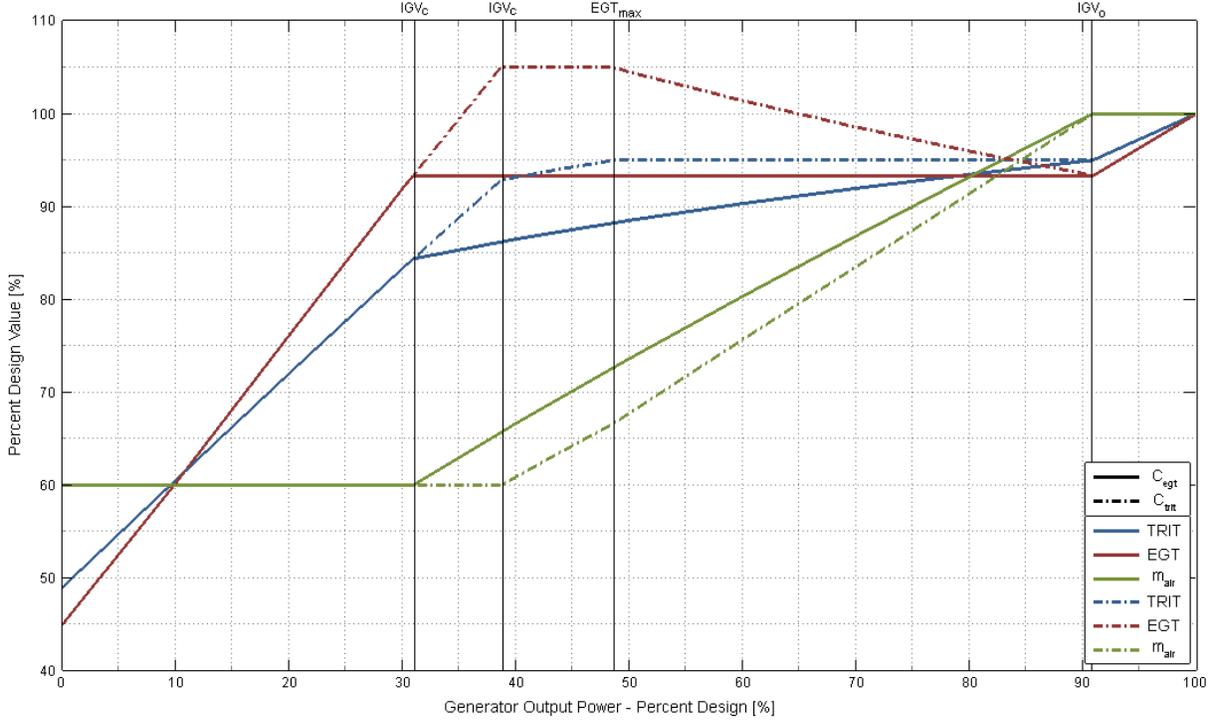


Figure 3: Engine Control Curve Impact on TRIT, EGT, M_{air}

For example, take the generic curve shown in Figure 3. If the user inputs that they wish to know what the part-load operating conditions of the engine were at 20% load, it is apparent that the conditions at 20% load cannot be directly solved given the full-load parameters.

Instead, the model must step through the model, first solving for the conditions at the inlet guide vane closure, then solving for the conditions at the location where the exhaust temperature reaches its maximum value, then to the point at which the inlet guide vanes are fully closed, and finally to the percent full-load power output desired by the user.

In the most general sense, the structure at which the logical statements are set up in pseudo-code as follows:

Listing 1: Logic Branching Pseudocode

```

1   if (percentLoad >= IGVopen) {
2       partload(percentLoad);
3   }
4
5   if (percentLoad < IGVopen && percentLoad > IGVclose) {
6       partload(IGVopen);
7
8       if (percentLoad >= EGTswitch) {
9           partload(percentLoad);
10          } else {

```

```

11             partload(EGTswitch);
12             partload(percentLoad);
13         }
14     }
15
16     if (percentLoad <= IGVclose) {
17         partload(IGVopen);
18         partload(EGTswitch);
19         partload(IGVclose);
20         partload(percentLoad);
21     }

```

At its core, this is simply how the logic branching works. Depending on where you are in the model, different conditions will be met, and the model modifies its solution method accordingly.

3 Conclusion

The objective of this project was, not only to figure out how to model gas turbine engines at part-load, but to be able to implement these part-load gas turbine models in computer programming code that would be entirely flexible while remaining computationally efficient.

The models were initially prototyped in MATLAB and Python, for their ease of use and their included functions such as matrix inversion in MATLAB, and add-on packages to Python such as *numpy* and *matplotlib*.

This was completed through the creation of many numerical solvers and independent solution functions using the C++ programming language. Included in these solvers were provisions for engine control curves. Two independent control curves were used, one with constant turbine rotor inlet temperature, or firing temperature, and one with constant exhaust temperature.

The assumptions used for control theory such as varying exhaust gas temperature, firing temperature, and air mass flow rate proved to generate curves very similar to those predicted by the manufacturer's detailed models. Also, assumptions for comparing the full-load model to the part-load model such as choked flow into the turbine and volumetric flow rate across the compressor proved to be accurate as compared to the limited curves provided by the manufacturers.

In order to solve for the part-load operating conditions while still maintaining consistency with the engine control curves, a method for stepping through the models called Logic Branching was used. This solution method ensured that the mathematics behind the physical laws governing the gas turbine engine meshed with the engineering technical intuition required to operate gas turbine engines.

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